# THE RELATION BETWEEN SPATIAL AND TEMPORAL EFFECTS OF DEFORMATION AND FRACTURE OF QUASI-BRITTLE MATERIALS

Chengzhi Qi[*a)], Mingyang Wang[**], Qihu Qian[**] and Jianjie Chen[***]

[*] *Beijing Research Center for Engineering Structures and New Materials, Beijing University of Civil Engineering and Architecture, Beijing , 100044, China;*

[**] *Civil Engineering Department, Nanjing Engineering Institute, Nanjing, 210007, China.*

[***] *Northwest Institute of Nuclear Technology, Xi'an, 710024, China*

Summary The relationship between spatial and temporal effects of deformation and fracture of quasi-brittle materials are studied from the viewpoint of structural hierarchy of materials for the first time. It is shown that the relationship between spatial and temporal effects of deformation and fracture of rock-like materials is determined by the finiteness of crack propagation velocity. Size effect may be looked at as the realization of the structural surface strength of small scale structural elements with the decrease of the sample size. The essence of strain rate effect is that because of the finiteness of crack propagation velocity at high strain rates the deformation and fracture process of small scale elements are activated before macro-fracture of the sample, the dynamic strength of material is the structural surface strength of the least activated elements before the macro-fracture of the sample as a whole.

## SIZE AND STRAIN RATE EFFECTS

Deformation and fracture of quasi-brittle materials, such as rock and concrete, possess temporal and spatial effects. Generally, the researches on temporal and spatial effects of deformation and fracture of quasi-brittle materials are mainly concentrated on size effect and strain rate effect of strength. For type I size effect an appropriate equation for description of dependence of strength $\sigma_N$ on sample size $D$ was given by Bazant et al [1]

$$\sigma_N(D) = f_r^{\infty}\left[\left(\frac{l_s}{D+l_s}\right)^{rn/m} + \frac{rD_b}{D+l_p}\right]^{1/r} \qquad (1)$$

where $f_r^{\infty}$ is the nominal strength of sample of very large size; $D_b, l_p, l_s, r, n, m$ are parameters of the model.

One of the significant temporal features of deformation and fracture of materials, including quasi-brittle materials, is the strain rate sensitivity of their strength. Based on the achievements of scientists in this area the writers of the present paper proposed a model with the competition between the coexisting thermally activated and macro-viscous mechanisms [2]

$$\sigma_Y = \frac{1}{\gamma}\left(U_0 + kT\ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right) + \frac{b(\dot{\varepsilon}/\dot{\varepsilon}_S)^n}{(\dot{\varepsilon}/\dot{\varepsilon}_S)^n + 1} \qquad (2)$$

where $U_0$ is the activation energy; $\gamma$ is the activation volume; $k$ is the Boltzmann's constant; $T$ is the absolute temperature; $\dot{\varepsilon}_0$ represents the maximum possible strain rate in the material with $\varepsilon_0$ being the limit of deformation at failure, $\tau_0$ being a temporal parameter (about $10^{-12}$ s); $b$, $\dot{\varepsilon}_S$ and $n$ are model parameters.

Until now the writers of the present paper have not found any works on investigation on the relationship between size and strain rate effects. Therefore we here will make a trial to study the relationship between temporal and spatial effects.

## THE RELATIONSHIP BETWEEN SIZE AND STRAIN RATE EFFECTS

The evolution equation for the residual stress deviator $\Delta s_{ij}^l$ in heterogeneities of size $l$ in quasi-brittle materials may be described by Maxwell model

$$d\Delta s_{ij}^l / dt = 2\rho c_s^2 \dot{e}_{ij} - v\Delta s_{ij}^l / l \qquad (3)$$

where $\dot{e}_{ij}$ is the residual strain rate deviator components; $\rho$ is the density of the medium; $v$ is the relaxation velocity; $c_s$ is the propagation velocity of the elastic shear wave.

The stationary solution of Eq.(3) is $\Delta s_{ij}^l = 2\rho c_s^2 \dot{e}_{ij} / v$ or in intensities of residual stress deviator $\Delta\sigma_I$ and residual strain rate intensity $\dot{\varepsilon}_I$ as

$$\Delta\sigma_I = 3\rho c_s^2 \dot{\varepsilon}_I / v \qquad (4)$$

If we denote the limit residual stress causing fracture of the body as $\sigma^*$, then we can determine the minimal size of heterogeneities corresponding to the limit residual stress $\sigma^*$:

$$l = \sigma^* v / \left(3\rho c_s^2 \dot{\varepsilon}\right) \qquad (5)$$

For plastically non-compressible quasi-brittle materials Poisson's ratio $\mu = 0.5$, and Young's elastic modulus is $E = 3\rho c_s^2$. Therefore Eq.(8) may be rewritten as

$$l = \sigma^* v / \left(3\rho c_s^2 \dot{\varepsilon}\right) = \varepsilon^* v / \dot{\varepsilon} = \tau^* v \qquad (6)$$

which denotes the propagation distance of crack at the moment of failure.

[a] Corresponding author. Email: qczbicea@yahoo.com.cn

If we replace the parameter $D$ in Eq.(1) by $l$ in Eq.（1）, we obtain

$$\sigma_N(D)= f_r^{\infty}\left[\left(\frac{l_s}{D+l_s}\right)^{rn/m}+\frac{rD_b}{D+l_p}\right]^{1/r}=f_r^{\infty}\left[\left(\frac{\dot{\varepsilon}/\dot{\varepsilon}_s}{\dot{\varepsilon}/\dot{\varepsilon}_s+1}\right)^{rn/m}+\frac{r(\dot{\varepsilon}/\dot{\varepsilon}_b)}{(\dot{\varepsilon}/\dot{\varepsilon}_p)+1}\right]^{1/r} \tag{7}$$

where parameters $\dot{\varepsilon},\dot{\varepsilon}_s,\dot{\varepsilon}_b$ and $\dot{\varepsilon}_p$ are strain rates corresponding to size parameters $D,l_s,D_b$ and $l_p$ respectively according to Eq.(5). It can be seen easily that Eqs.（7）and（2）have the similar asymptotes at low and high strain rates (or at large and small size scales respectively) which reveal the intrinsic relationship between size and strain rate effects.

Propagation velocity $v$ of crack depends on loading conditions and material properties. The dependence of fracture propagation rate on the stress intensity factor in mode I may be approximated by trimodal behavior [3], which is shown in Fig.1. In region I, the rate of stress corrosion reaction control the velocity of crack growth. Region II is mainly determined by the rate of transport of reactive species to crack tips. In region III the velocity of crack growth increases drastically up to failure and is relatively independent of the chemical environment and is controlled by mechanical rupture. Therefore there are two characteristic crack propagation velocities: the growth velocity corresponding to plateau II $v_{sub}$ and limit crack growth velocity $v_C$ .

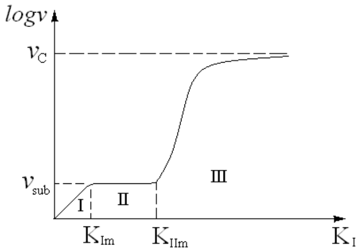

Fig.1 The dependence of crack propagation velocity on stress intensity factor

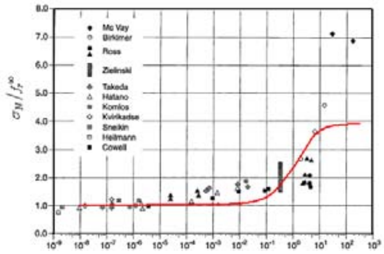
Fig. 2 Modeling of strain rate effect by Eq. (8)

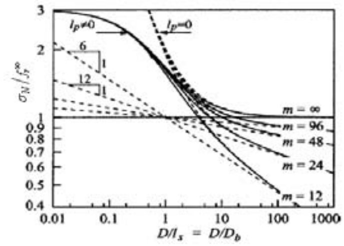

Fig.3 The size effect of concrete (after Bazant et al [1])

For $v_{sub}\approx 2\times10^{-8}m/s$, $v_c=800m/s$, $\varepsilon^*=1.5\times10^{-4}$, $D_b=0.28m$, $r=1$, $l_p=D_b/3=0.093m$, $l_s=D_b$, $r=1$, $m=48$, $n=2$. We have $\dot{\varepsilon}_s=\varepsilon^* v_{sub}/l_s=1.07\times10^{-11}/s;\dot{\varepsilon}_p=\varepsilon^* v_C/l_p=1.3/s;\dot{\varepsilon}_b=\varepsilon^* v_C/D_b=0.43/s$ .Eq. (7) may be rewritten as

$$DIF=\frac{\sigma_N(D)}{f_r^{\infty}}=\left[\left(\frac{\dot{\varepsilon}/\dot{\varepsilon}_s}{\dot{\varepsilon}/\dot{\varepsilon}_s+1}\right)^{rn/m}+\frac{r(\dot{\varepsilon}/\dot{\varepsilon}_b)}{(\dot{\varepsilon}/\dot{\varepsilon}_p)+1}\right]^{1/r}=\left[\left(\frac{0.93\times10^{11}\dot{\varepsilon}}{0.93\times10^{11}\dot{\varepsilon}+1}\right)^{1/24}+\frac{3\dot{\varepsilon}}{\dot{\varepsilon}+1.3}\right] \tag{8}$$

where *DIF* denotes the dynamic increase factor of material strength. For experimental data of tensile strength of concrete shown in Fig.2, the numerical results by Eq. (8) is represented by the solid line. It is shown from Fig.2 that Eq. (8) describes strain rate sensitivity very well.

It can be seen from Figs.2 and 3 that the strength increase factor of concrete for size and strain rate effects is in the range of 3~5 (the experimental data for dynamic *DIF* of tensile strength of concrete higher than 5 are rare and not reliable, for compression strength of concrete the dynamic *DIF* is generally lower than 3), this result is not accidental, but is of the expression of the relationship between spatial and temporal effects of deformation and fracture of quasi-brittle materials.

## CONCLUSION

The study on the relationship between spatial and temporal effects of quasibrittle materials in the present paper shows that the relationship between spatial and temporal effects of deformation and fracture of quasibrittle materials is determined by the finitness of crack propagation velocity. Size effect may be looked at as the realization of the structural surface strength of small scale structural elements with the decrease of the sample size. The essence of strain rate effect is that because of the finitness of crack propagation velocity at high strain rates the deformation and fracture process of small scale elements are activated before macro-fracture of the sample, the dynamic strength of material is the structural surface strength of the activated least scale elements before the macro-fracture of the sample as a whole.

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