HARDENING COHESIVE ZONE MODEL FOR SIZE EFFECTS IN THE NECKING AND DUCTILE FRACTURE OF METALS

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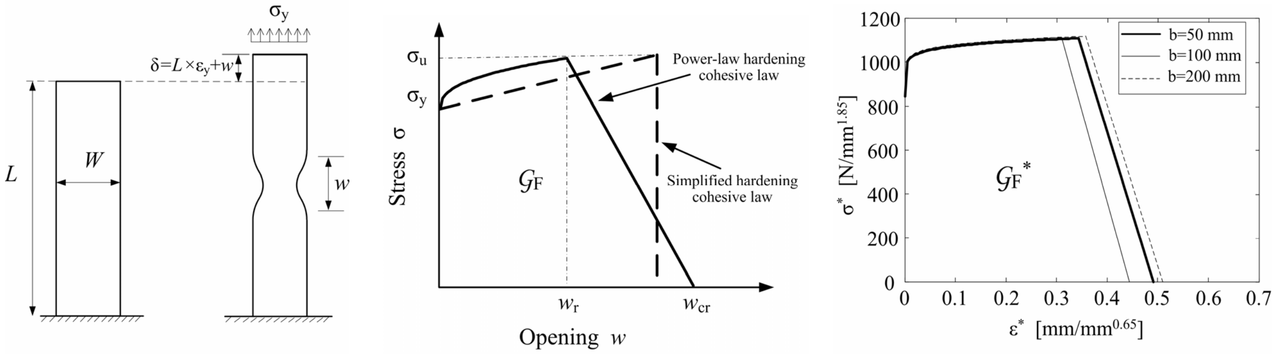
Summary A new Hardening Cohesive/Overlapping Zone Model is proposed for ductile fracture phenomena in metallic materials. The strain localization characterising the hardening and softening stages is modelled by means of stress vs. displacement (fictitious opening for tension and interpenetration for compression) constitutive laws. Such a model is applied to study the size effects in three-point-bending and compact tension tests. The fractal approach is also considered to experimentally obtain scale-independent hardening cohesive/overlapping laws.

ANALYTICAL AND NUMERICAL FORMULATION

Hardening Cohesive Zone Model

Applications of the Cohesive Zone Model to ductile fracture in metals are rather common today [1]. However, such a model is generally used to describe only the post-peak softening regime, whereas stress vs. strain constitutive laws are adopted for the hardening phase and the corresponding localized plastic dissipations. In this context, the main novelty of the present study is the application of a Hardening Cohesive Zone Model since the beginning of the hardening phase, up to complete failure. This means that the cohesive process zone is representative of the region where the localized plastic dissipation in the bulk and the crack formation and propagation take place. With reference to a uniaxial tensile test, when the elastic limit is overcome, the deformation starts to localize within a limited specimen portion, well-known as necking zone, and the total elongation is given by the localized contribution, w, and the elastic one, given by the bulk material (Fig. 1a). The behaviour of the necking region is described by a stress vs. displacement constitutive law (solid line in Fig. 1b), characterized by a generic hardening branch up to the limit value w_r. As the necking deformation proceeds, microvoids enlargement and coalescence lead to the softening behaviour, with $\sigma < \sigma_u$ and $w_r < w < w_{cr}$. The localized contribution to the elongation still increases, whereas the elastic one decreases according to the loading decrement. When $w > w_{cr}$, the reacting stress vanishes, and the material in the necking zone is completely failed. In the present study, the same constitutive law is assumed for compression, where localized plastic flows develop into slip-lines. It will be referred to as Hardening Overlapping Zone Model.



(a) (b) (c)

Fig. 1. Localized necking deformation in uniaxial tension (a); hardening cohesive law for the necking region (b); fractal cohesive law (c).

Numerical algorithm

The numerical algorithm proposed by Carpinteri and co-workers [2] for the integrated Cohesive/Overlapping Crack Model, is herein adopted to describe the fracturing and the plastic phenomena taking place in notched three-point-bending and compact test specimens. In both cases, the sample is subdivided into two elastic parts connected along the mid-span cross-section by n pairs of nodes, where the elastic stresses in the ligament, and the cohesive and overlapping stresses in the process zones are replaced by equivalent nodal forces. Such forces can be expressed as a function of the nodal relative displacements, $\{w\}$, and the applied load, P, by means of elastic coefficients of influence preliminary determined by a finite element analysis:

$$\{F\} = [K_w]\{w\} + \{K_P\}P \tag{1}$$

Equation (1) constitutes a linear algebraic system of (n) equations in ($2n$+1) unknowns, {F}, $\{w\}$ and P. Hence, to solve the problem and determine the load-deflection diagram, additional (n+1) equations have to be introduced by considering the constitutive laws in tension and compression, and a strength criterion for governing the propagation processes. At each step, the driving parameter during the simulation is either the fictitious crack length or the fictitious

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overlapping extension on the basis of the above strength criterion (see [2] for more details). Finally, at each step, the displacement is computed as a function of $\{w\}$ and P by means of elastic coefficients of influence.

Limit Analysis

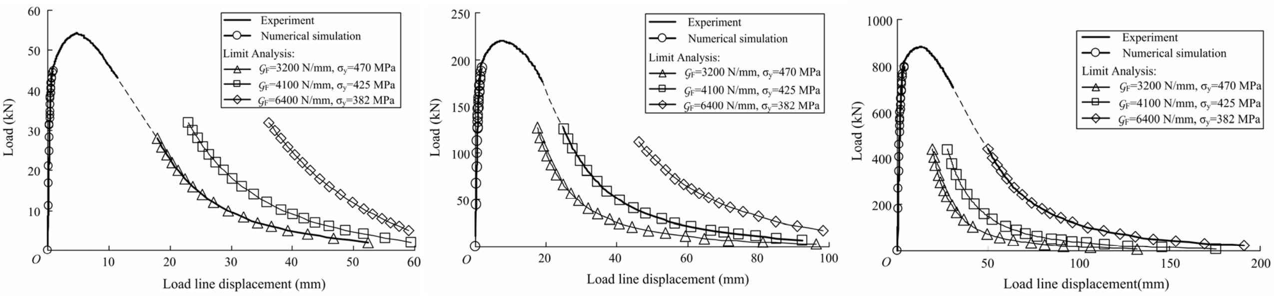
The numerical procedure proposed in the previous section terminates when the two fictitious tips converge to the same node. On the other hand, the entire post-peak load vs. deflection curve is relevant for the analysis of ductile-to-brittle transitions. Therefore, a limit analysis approach based on the proposed constitutive laws is applied to obtain an asymptotic approximation to the post-peak response. From a practical point of view, the specimens, both the beam and the compact samples, are considered as constituted by two symmetrical rigid parts, connected by a hinge in the mid-span cross-section. In this limit stage, the equilibrium of each part is ensured by the external load, the support reaction and the cohesive/overlapping forces applied along the mid-span cross-section. By increasing the rotation around the hinge, the process zones reduce and, consequently, the load-carrying capacity decreases. In this approach, the assumption of a stress–displacement law having a linear-hardening with subsequent vertical drop (dashed curve in Fig. 1b) permits a simple closed-form analytical solution to be obtained.

COMPARISON OF PREDICTIONS VERSUS EXPERIMENTAL RESULTS

The proposed numerical and analytical approach has been used to predict the behaviour of three-point-bending and compact tension tests available in the literature. In particular, the experimental tests taken from the Euro-Fracture Dataset [3] are worth of interest, since they refer to compact tensile specimens with different sizes (width W = 50, 100 and 200 mm). In this case, the fractal approach to scale effects proposed and discussed widely by Carpinteri and co-workers [7, 8] for quasi-brittle materials, has been adopted. Also in metals, in fact, experimental evidences exist of size effects on the ultimate strength [4], and of strain localization and energy dissipation patterns having fractal character [5, 6]. Accordingly, the following scaling laws have been assumed for the cohesive/overlapping parameters:

$$\sigma_{\mathrm{y}} = \sigma_{\mathrm{y}}^{*} W^{-d_{\sigma}}\,; \quad \mathcal{G}_{\mathrm{F}} = \mathcal{G}_{\mathrm{F}}^{*} W^{d_{\mathcal{G}}}\,; \quad w_{\mathrm{cr}} = \varepsilon_{\mathrm{cr}}^{*} W^{(1-d_{\varepsilon})}\,, \tag{2a,b,c}$$

where σ_{y}^{*}, $\mathcal{G}_{\mathrm{F}}^{*}$ and $\varepsilon_{\mathrm{cr}}^{*}$ are the true scale invariant properties, having anomalous physical dimensions, and W is the representative specimen dimension. The numerical simulations and limit analysis predictions obtained by assuming scale-dependent constitutive laws are shown in Fig. 2. In particular, the mechanical properties reported in ref. [3] have been assigned to specimen 1T, which has the dimensions closer to those of the tensile specimen used to get them. Then, a good continuity between experiments and limit analysis is obtained if the following power-law exponents are assumed for the parameters' scaling laws: $d_{\mathcal{G}}$ = 0.50, d_{σ} = 0.15 and d_{ε} = 0.35 (the values of $\mathcal{G}_{\mathrm{F}}$ and σ_{y} for the different specimen sizes are reported in Fig. 2). Finally, the fractal cohesive laws have been derived by applying the inverse of Eqs. (2) (see Fig. 1c). Such curves are not exactly coincident, although they tend to converge to a unique one, confirming the validity of the fractal approach in determining size-independent constitutive laws. The average values for the fractal parameters are: $\sigma_{\mathrm{y}}^{*} = 848.50\mathrm{N/mm}^{1.85}$, $\sigma_{\mathrm{u}}^{*} = 1112.00\mathrm{N/mm}^{1.85}$, $\varepsilon_{\mathrm{y}}^{*} = 0.48\mathrm{mm/mm}^{0.65}$, and $\mathcal{G}_{\mathrm{F}}^{*} = 438.00\mathrm{N/mm}^{1.50}$.



(a) Specimen 1T (W = 50 mm) (b) Specimen 2T (W = 100 mm) (c) Specimen 4T (W = 200 mm)

Fig. 2. Load–displacement curves for CT specimens of different sizes. Experimental results from ref. [3].

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