AN INTERACTION ENERGY INTEGRAL METHOD FOR AN INTERFACE CRACK BETWEEN TWO NONHOMOGENEOUS MATERIALS UNDER THERMAL LOADING

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Summary The interface crack between two nonhomogeneous materials subjected to steady thermal loading is investigated. A modified interaction energy integral method (IEIM) is developed to obtain the mixed-mode thermal stress intensity factors (TSIFs). The IEIM is proved to be domain-independent. Then the modified IEIM is used to solve the thermal interfacial fracture problems. Results show the method is effective and efficient.

INTERACTION INTEGRAL FOR AN INTERFACE CRACK BETWEEN TWO MATERIALS

A path-independent form of interaction energy integral is derived for thermal crack problems in this section. The traditional J-integral (Rice, 1968) is given by:

$$J = \lim_{\Gamma_0 \to 0} \int_{\Gamma_0} (W\delta_{1j} - \sigma_{ij} u_{i,1}) n_j d\Gamma \tag{1}$$

where n_j is the outward normal vector to the contour Γ_0, which starts from a point on the lower crack face and ends at another point on the upper crack face, as shown in Fig. 1.

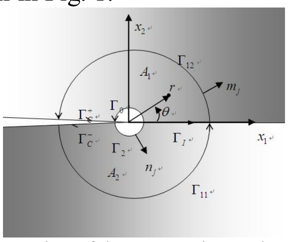

Fig. 1. Schematic illustration of the contour integrals and domain integrals.

And δ_{ij} is the Kronecker delta. In Eq. (1), W denotes the strain energy density expressed as

$$W = \frac{1}{2}\sigma_{ij}\varepsilon_{ij}^{m} = \frac{1}{2}\sigma_{ij}(\varepsilon_{ij}^{t} - \varepsilon_{ij}^{th}) \tag{2}$$

Here ε_{ij}^{m} is the mechanical strain, ε_{ij}^{t} denotes the total strain, $\varepsilon_{ij}^{th} = \alpha\Delta T\delta_{ij}$ refers to thermal strain, α represents the thermal expansion coefficient and $\Delta T = T - T_0$ denotes temperature change with T_0 as the initial temperature. The interaction energy integral can be derived as (Kim and Paulino, 2005)

$$I = \lim_{\Gamma_0 \to 0} \oint_{\Gamma_2} (\sigma_{ij}^{aux} u_{i,1} + \sigma_{ij} u_{i,1}^{aux} - \sigma_{ik}^{aux} \varepsilon_{ik}^{m} \delta_{1j}) m_j q d\Gamma \tag{3}$$

By using the divergence theorem, an equivalent domain integral can be deduced from Eq. (3) as

$$I = I_1 + I_2 + I_{\Gamma_I} \tag{4}$$

Here,

$$I_1 = \int_A (\sigma_{ij}^{aux} u_{i,1} + \sigma_{ij} u_{i,1}^{aux} - \sigma_{ik}^{aux} \varepsilon_{ik}^{m} \delta_{1j}) q_{,j} dA \tag{5}$$

$$I_2 = \int_A (\sigma_{ij}^{aux} u_{i,j1} + \sigma_{ij} u_{i,j1}^{aux} - \sigma_{ij,1}^{aux} \varepsilon_{ij}^{m} - \sigma_{ij}^{aux} \varepsilon_{ij,1}^{m}) q dA \tag{6}$$

$$I_{\Gamma_I} = \int_{\Gamma_I} \left\{ (\sigma_{ij}^{aux} u_{i,1} + \sigma_{ij} u_{i,1}^{aux} - \sigma_{ik}^{aux} \varepsilon_{ik}^{m} \delta_{1j})^{②} - (\sigma_{ij}^{aux} u_{i,1} + \sigma_{ij} u_{i,1}^{aux} - \sigma_{ik}^{aux} \varepsilon_{ik}^{m} \delta_{1j})^{①} \right\} m_j q d\Gamma \tag{7}$$

On the interface Γ_I, $m_1 = 0$ and $m_2 = -1$ as shown in Fig. 1. According to Smelser and Gurtin (1977) and the definitions of the auxiliary fields, the stress and displacement conditions on the interface Γ_I can be written as

$$\begin{cases} \sigma_{2i}^{①} = \sigma_{2i}^{②} = \sigma_{2i}, & u_i^{①} = u_i^{②} = u_i, & u_{i,1}^{①} = u_{i,1}^{②} = u_{i,1}, \\ \sigma_{2i}^{aux①} = \sigma_{2i}^{aux②} = \sigma_{2i}^{aux}, & u_i^{aux①} = u_i^{aux②} = u_i^{aux}, & u_{i,1}^{aux①} = u_{i,1}^{aux②} = u_{i,1}^{aux} \end{cases} \quad (i = 1, 2) \tag{8}$$

Substituting Eq.(8) and m_j into the integral I_{Γ_I}, we have

$$I_{\Gamma_I} = \int_{\Gamma_I} \left\{ \left[(\sigma_{ij}^{②} - \sigma_{ij}^{①}) u_{i,1}^{aux} + (\sigma_{ij}^{aux②} - \sigma_{ij}^{aux①}) u_{i,1} \right] m_j - \left[(\sigma_{ij}^{aux}\varepsilon_{ij})^{②} - (\sigma_{ij}^{aux}\varepsilon_{ij})^{①} \right] m_1 \right\} q d\Gamma = 0 \tag{9}$$

According to the relationship of displacement and strain in elastic mechanics, one may write the first term $\sigma_{ij}^{aux} u_{i,j1}$ as

$$\sigma_{ij}^{aux} u_{i,j1} = \frac{1}{2}\sigma_{ij}^{aux}(u_{i,j1} + u_{j,i1}) = \sigma_{ij}^{aux}\varepsilon_{ij,1}^{t} = \sigma_{ij}^{aux}(\varepsilon_{ij,1}^{m} + \varepsilon_{ij,1}^{th}) \tag{10}$$

So

$$I_2 = \int_A (\sigma_{ij} u_{i,j1}^{aux} - \sigma_{ij,1}^{aux} \varepsilon_{ij}^{m} + \sigma_{ij}^{aux} \varepsilon_{ij,1}^{th}) q dA \tag{11}$$

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For further derivation, an extra strain field $\varepsilon_{ij}^{aux0} = S_{ijkl}^{tip}\sigma_{kl}^{aux}$ is introduced and the formulation $\varepsilon_{ij}^{aux0} = \frac{1}{2}(u_{i,j}^{aux} + u_{j,i}^{aux})$ can be got (Yu et al., 2009). Where S_{ijkl}^{tip} is a compliance tensor at the crack tip. Then, the first integrand in Eq. (11) can be written as

$$\sigma_{ij}u_{i,j1}^{aux} = \sigma_{ij}\varepsilon_{ij,1}^{aux0} = \sigma_{ij}S_{ijkl}^{tip}\sigma_{kl,1}^{aux} \tag{12}$$

Finally, the interaction energy integral for the thermal interfacial problem of nonhomogeneous material is given by

$$\begin{aligned} I = &\int_A (\sigma_{ij}^{aux}u_{i,1} + \sigma_{ij}u_{i,1}^{aux} - \sigma_{ik}^{aux}\varepsilon_{ik}^{m}\delta_{1j})q_{,j}dA \\ &+ \int_A (\sigma_{ij}(S_{ijkl}^{tip} - S_{ijkl}(x))\sigma_{kl,1}^{aux})qdA + I^{th} \end{aligned} \tag{13}$$

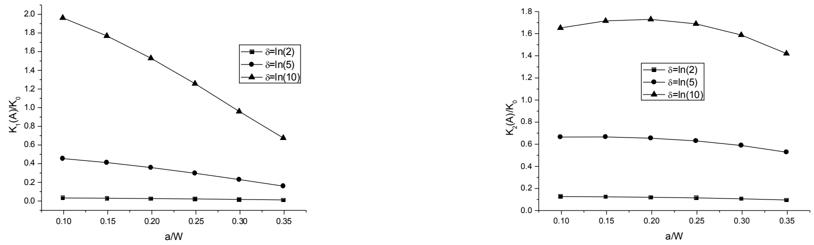
where I^{th} is a new term in the interaction energy integral for the thermal interfacial problem. According to the relation $\varepsilon_{ij}^{th} = \alpha\Delta T\delta_{ij}$, I^{th} can be expressed as

$$I^{th} = \int_A (\sigma_{ij}^{aux}\varepsilon_{ij,1}^{th})qdA = \int_A \sigma_{ii}^{aux}[\alpha_{,1}(T - T_0) + \alpha T_{,1}]qdA \tag{14}$$

Due to the reason that the domain of the integral is chosen arbitrarily around the crack-tip in the above derivation process, the interaction energy integral for thermal fracture interfacial problems is domain-independent.

NUMERICAL EXAMPLES

An interface crack between two nonhomogeneous elastic semi-infinite planes is considered. The specimen is a plate with length L and width W which contains a center interface crack of length $2a$. The displacement boundary conditions are prescribed such that $u_2 = 0$ along the bottom edge. The uniform temperature change $T_1 = T_2 = 10T_0$ are adopted. Young's moduli and the thermal expansion coefficients of Material 1 and Material 2 are E_1, α_1 and E_2, α_2, respectively, which are defined as $E_1 = E_0e^{\delta x_1}, E_2 = E_0e^{-\delta x_1}, \alpha_1 = \alpha_0e^{\delta x_1}, \alpha_2 = \alpha_0e^{-\delta x_1}$. ν_1 and ν_2 are the corresponding Poisson's ratios, respectively. The following data are used for numerical analysis: $L = 2$; $W = 1$; $E_0 = 1000$; $\alpha_0 = 1\times10^{-5}$; $\delta = [\ln(2), \ln(5), \ln(10)]$; $\nu_1 = \nu_2 = 0.3$; $K_0 = E_0\alpha_0(T_1 - T_0)\sqrt{\pi a}/(1-\nu)$; generalized plane strain. The results of the variations of normalized TSIFs with the crack length a/W are shown in Fig. 2.



(a) (b)

Fig. 2 Normalized SIFs of a center interface crack for two nonhomogeneous materials : (a) K_1/K_0; (b) K_2/K_0.

It can be seen that the normalized TSIFs K_1/K_0 decrease with the increasing of a/W. However, K_2/K_0 increase slightly first and then, decrease with the increasing of a/W. The magnitude of normalized TSIFs increases with the increasing of parameter δ.

CONCLUSIONS

In this paper, a modified IEIM is proposed to solve the interfacial crack problem for two nonhomogeneous materials. The implementation of the interaction energy integral is combined with the XFEM and the results show that the method is effective and efficient for the nonhomogeneous materials.

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