ANALYSIS OF COMPLEX CRACK PROBLEMS BY THE POLYGONAL NUMERICAL MANIFOLD METHOD

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Summary The numerical manifold method (NMM) is very suitable for crack problems due to the use of two cover systems. At the same time, the *n*-sided elements (*n*>4) are very attractive because of their great flexibility in meshing and high accuracy. At present, the NMM is applied to solve complex crack problems on regular hexagonal elements. Our results show that the accuracy of the proposed method is higher than that on regular triangular and quadrilateral elements.

INTRODUCITON

The NMM, originally proposed by Shi in [1], is very suitable for crack problems and has been widely applied to tackle crack problems. Ma et al [2] investigated the multiple crack problems by incorporating the singular physical covers; Zhang et al [3] studied the complex crack growth problems. Kurumatani and Terada [4] focused on the crack simulations for quasi-brittle heterogeneous solids. Among the previous work, triangular elements (e.g. in [2, 3]) and quadrilateral elements (e.g., in [4]) are mainly used. To make full use of the special feature of the *n*-sided (*n*>4) elements, in the present paper, the NMM, combined with the regular hexagonal mathematical elements, are applied to solve complex crack problems.

CRACK MODELING WITH THE POLYGONAL NMM

Brief of the polygonal NMM

In the NMM, there are two cover systems (the mathematical cover system and the physical cover system) and five main concepts (mathematical element, mathematical cover, physical cover, manifold elements and weight function). To solve a concerned problem by the NMM, the mathematical cover system, which is the union of mathematical covers that composed of different mathematical elements, is first taken to cover the whole physical domain without considering any physical features such as material interfaces, cracks and boundaries. Then, the physical cover system, formed by a series of physical covers, is generated through the intersection of mathematical cover system and physical domain. Finally, the manifold elements are collected through the common region of several physical covers. On each mathematical cover a weight function is defined which satisfies the property of the partition of unity. In the present, the weight functions for *n*-sided mathematical elements with *n* =3, 4 and 6 is expressed by the Wachspress basis [5].

Crack modeling by the NMM

For general crack problems, the displacement approximation on a manifold element *ME* can be expressed as [2, 3]

$$\mathbf{u}^h(\mathbf{x}) = \sum_{i=1}^{NP} w_i(\mathbf{x}) \cdot \mathbf{u}_i(\mathbf{x}) + \sum_{j=1}^{NPT} w_j(\mathbf{x}) \cdot \hat{\mathbf{u}}_j(\mathbf{x}) \tag{1}$$

where *NP* and *NPT* are respectively the number of physical covers and tip physical covers associated with *ME*; $w_i(\mathbf{x})$ is the weight function and $\mathbf{u}_i(\mathbf{x})$ is the cover function as

$$\mathbf{u}_i(\mathbf{x}) = \mathbf{P}(\mathbf{x}) \cdot \mathbf{a}_i \tag{2}$$

where $\mathbf{P}(\mathbf{x})$ is the matrix of polynomial bases and in 2-D setting

$$\mathbf{P}(\mathbf{x}) = \begin{bmatrix} 1 & 0 & x & 0 & y & 0 & \cdots & 0 \\ 0 & 1 & 0 & x & 0 & y & 0 & \cdots \end{bmatrix} \tag{3}$$

and $\mathbf{a}_i$ is the vector of unknowns. $\hat{\mathbf{u}}_j(\mathbf{x})$ is the additional cover function as

$$\hat{\mathbf{u}}_j = \mathbf{\Phi} \cdot \mathbf{b}_j \tag{4}$$

for singular physical covers that containing crack tip. $\mathbf{b}_j$ is the array of additional unknowns and $\mathbf{\Phi}$ is the matrix of singular bases as

$$\mathbf{\Phi} = \begin{bmatrix} \Phi_1 & 0 & \Phi_2 & 0 & \Phi_3 & 0 & \Phi_4 & 0 \\ 0 & \Phi_1 & 0 & \Phi_2 & 0 & \Phi_3 & 0 & \Phi_4 \end{bmatrix} \tag{5}$$

with

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$$[\Phi_1,\Phi_2,\Phi_3,\Phi_4]=\left[\sqrt{r}\sin\frac{\theta}{2},\sqrt{r}\cos\frac{\theta}{2},\sqrt{r}\sin\theta\sin\frac{\theta}{2},\sqrt{r}\sin\theta\cos\frac{\theta}{2}\right] \tag{6}$$

where (r,θ) are the polar coordinates in the local coordinates system with the origin at the crack tip.

NUMERICAL EXAMPLE

As shown in Fig. 1, a square plate with a crucifix crack under bi-axial tension is analyzed. The stress intensity factor (SIF) at point A is calculated. Mathematical cover systems with different cover size h (the diameter of the circumscribed circle of mathematical element) are used to cover the whole plate (the discretized plate when h=0.09W is shown in Fig. 2). The normalized mode I SIF by the reference solution in [6] is listed in Table 1 together with the results on quadrilateral and triangular elements, from which we can conclude that the results on the regular hexagonal elements tend to the reference solution with the decrease of the cover size; in addition, we can also find that the accuracy on the hexagonal elements is the best while those on the triangular elements is the worst.

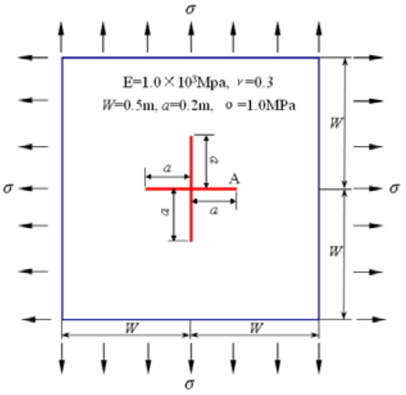
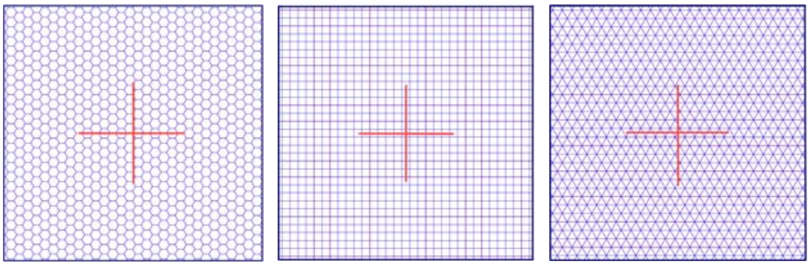


Fig.1. A square plate with a crucifix crack under tension



(a) by hexagonal elements (b) by quadrilateral elements (c) by triangular elements

Fig. 2. Discretized plate when h=0.09w

Table 1. Normalized SIF for different mathematical elements

Mathematical elements	Cover size h		
	0.24W	0.14W	0.09W
Regular hexagon	1.056	1.018	1.003
Regular quadrilateral	1.067	1.029	1.007
Regular triangle	1.095	1.055	1.014

CONCLUSIONS

Due to the independence of physical domain and the mathematical cover system, fully regular mathematical elements can be applied in the NMM. In the present work, the NMM on regular hexagonal mathematical elements is applied to solve complex crack problems. At the same time, the accuracy of the NMM on different mathematical elements is also investigated and the results show that the regular hexagonal element is the best and the quadrilateral element is better than the triangular one.

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