DELAMINATION BETWEEN A UNIDIRECTIONAL AND WOVEN COMPOSITE

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Summary A delamination between a unidirectional fiber reinforced carbon/epoxy material with the fibers in the 90°-direction and a balanced weave with fibers in the $0^\circ/90^\circ$ directions is considered (see Fig. 1). The first term of the asymptotic solution for the stress and displacement fields are found. These are used in two methods for determining the stress intensity factors, the energy release rate and phase angles using finite element results. Sample solutions are presented.

ASYMPTOTIC SOLUTIONS

The first term of the asymptotic solution is determined by means of the Stroh [1] and Lekhnitskii [2] formalisms. It is found that there are two singular solutions, one in which the singularity is square-root, oscillatory, and one in which it is square-root. For both cases, the displacement field $\boldsymbol{u}$ and stress function $\boldsymbol{\phi}$ for each material are found. The in-plane and out-of-plane fields decouple. Note that the upper material is transversely isotropic with the symmetry plane $x_3 = 0$ (see Fig. 1). It has five independent mechanical properties. The lower material is tetragonal with six independent mechanical properties.
The in-plane stress field is given by

$$\sigma_{\alpha\beta}^{(k)} = \frac{1}{\sqrt{2\pi r}} \left[\Re\left(K r^{i\epsilon}\right)\ {}_k\Sigma_{\alpha\beta}^{(1)}(\theta) + \Im\left(K r^{i\epsilon}\right)\ {}_k\Sigma_{\alpha\beta}^{(2)}(\theta) \right] \ ; \tag{1}$$

whereas, the out-of-plane stress field is found as

$$\sigma_{\alpha 3}^{(k)} = \frac{K_{III}}{\sqrt{2\pi r}}\, {}_k\Sigma_{\alpha 3}^{(III)}(\theta) \ . \tag{2}$$

In eqs. (1) and (2) $\alpha, \beta = 1, 2$, and $k = 1, 2$ denotes the upper and lower materials, respectively. The symbols $\Re$ and $\Im$ denote the real and imaginary part of the quantity in parentheses, respectively. The oscillatory parameter ε is given by

$$\varepsilon = \frac{1}{\pi} \tanh^{-1} \beta = \frac{1}{2\pi} \ln \frac{1+\beta}{1-\beta} \ . \tag{3}$$

The expression for β may be found from the Barnett-Lotte tensors as

$$0 \le \beta = \left\{ -\frac{1}{2} \mathrm{tr}(\breve{\mathbf{S}}^2) \right\}^{\frac{1}{2}} < 1 \ . \tag{4}$$

The 3×3 matrix

$$\breve{\mathbf{S}} = \boldsymbol{D}^{-1}\boldsymbol{W}, \tag{5}$$

$$\boldsymbol{D} = \boldsymbol{L}_1^{-1} + \boldsymbol{L}_2^{-1}, \tag{6}$$

$$\boldsymbol{W} = \boldsymbol{S}_1\boldsymbol{L}_1^{-1} - \boldsymbol{S}_2\boldsymbol{L}_2^{-1} \tag{7}$$

and $\mathrm{tr}(\cdot)$ is the trace of the matrix in parentheses. The tensors $\boldsymbol{S}_k$ and $\boldsymbol{L}_k$ ($k = 1, 2$ represents the upper and lower materials, respectively), known as the Barnett-Lothe tensors, are real and related to the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ by

$$-\boldsymbol{A}_k\boldsymbol{B}_k^{-1} = \boldsymbol{S}_k\boldsymbol{L}_k^{-1} + i\boldsymbol{L}_k^{-1}, \tag{8}$$

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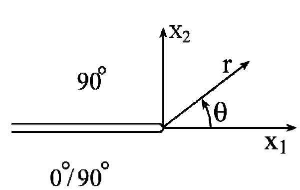

Figure 1. Delamination front coordinates.

where there is no summation on k and $i = \sqrt{-1}$. The 3×3 matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are dependent on the mechanical properties of the materials. The crack tip, polar coordinates r and θ are shown in Fig. 1. The complex stress intensity factor in eq. (1) is given by

$$K = K_1 + iK_2 \tag{9}$$

where K_1 and K_2 are real and, respectively, the modes 1 and 2 stress intensity factors; K_{III} is the mode III stress intensity factor. The nondimensional functions ${}_k\Sigma^{(1)}_{\alpha\beta}(\theta)$, ${}_k\Sigma^{(2)}_{\alpha\beta}(\theta)$ and ${}_k\Sigma^{(III)}_{\alpha\beta}(\theta)$ depend on the coordinate θ and the mechanical properties; they are associated with modes 1, 2 and III, respectively.
It is possible to show that the tractions along the interface may be related to the stress intensity factors as

$$\left(\sigma_{yy} + i\sqrt{\frac{D_{11}}{D_{22}}}\sigma_{xy}\right)\Bigg|_{\theta=0} = \frac{Kr^{i\varepsilon}}{\sqrt{2\pi r}}\ , \qquad\qquad \sigma_{yz}\Bigg|_{\theta=0} = \frac{K_{III}}{\sqrt{2\pi r}}\ . \tag{10}$$

General expressions for the in-plane displacement field may be written as

$$u^{(k)}_\alpha = \frac{1}{(1+4\varepsilon^2)H_k \cosh \pi\varepsilon}\sqrt{\frac{r}{2\pi}}\left[\Re\left(Kr^{i\varepsilon}\right)\ {}_kU^{(1)}_\alpha(\theta) + \Im\left(Kr^{i\varepsilon}\right)\ {}_kU^{(2)}_\alpha(\theta)\right]\ ; \tag{11}$$

whereas, the out-of-plane displacement is given by

$$u^{(k)}_3 = \frac{2}{\tilde{H}_k}\sqrt{\frac{r}{2\pi}}K_{III}\ {}_kU^{(III)}_3(\theta)\ . \tag{12}$$

In eqs. (11) and (12), H_k and $\tilde{H}_k$ have units of stress, ${}_kU^{(1)}_\alpha(\theta)$, ${}_kU^{(2)}_\alpha(\theta)$ and ${}_kU^{(III)}_\alpha(\theta)$ are nondimensional functions of θ and the mechanical properties and associated, respectively, with modes 1, 2 and III.
The displacement jump across the crack faces near the crack tip is found as

$$\Delta u_2 + i\sqrt{\frac{D_{22}}{D_{11}}}\Delta u_1 = \frac{2D_{22}}{(1+2i\varepsilon)\cosh \pi\varepsilon}\sqrt{\frac{r}{2\pi}}Kr^{i\varepsilon}\ , \qquad\qquad \Delta u_3 = 2D_{33}\sqrt{\frac{r}{2\pi}}K_{III} \tag{13}$$

where D_{11}, D_{22} and D_{33} are diagonal elements of the matrix $\boldsymbol{D}$ in eq. (6). In addition

$$\Delta u_i = u_i(r, \theta = \pi) - u_i(r, \theta = -\pi)\ . \tag{14}$$

Other interfaces have been studied in the past. This the first time in which a woven composite has been considered.

METHODS FOR EXTRACTING STRESS INTENSITY FACTORS

Stress intensity factors are calculated by two methods. First of all, finite element analyses are carried out to determine the displacement field. Then the displacement extrapolation method and the three-dimensional, interaction energy or M-integral are used to determine the K-values along the crack front. These methods utilize the asymptotic solutions presented in the previous section.

SAMPLE PROBLEMS

Several sample problems are solved for the interface considered in the study. Path independence of the M-integral, as well as solution convergence are considered.

CONCLUSIONS

An interface delamination between a unidirectional fiber reinforced carbon/epoxy material with the fibers in the $90°$-direction and a balanced weave with fibers in the $0°/90°$ directions is considered. Two methods for calculating stress intensity factors are used with results produced for several problems.

References

[1] Stroh, A.N.: Dislocations and Cracks in Anisotropic Elasticity. *Philosophical Magazine* **3**:625–646, 1958.
[2] Lekhnitskii, S.G.: Theory of Elasticity of an Anisotropic Body, Holden-Day, San Francisco, translated by P. Fern, 1950, in Russian; 1963, in English.