MECHANICAL PROPERTIES OF CELLULAR STRUCTURES WITH RIGID SQUARE ROTATIONS

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Summary The proposed motion structures with D_8 symmetry have two types of transformations with the rigid square rotations. One is the transformation into square cells which are tilted at a 45° (Motion I) and the other is the transformation into square cells (Motion II). We discuss about the elastic stiffness of the proposed structure, which is drastically changed during Motion I or Motion II.

INTRODUCTION

Cellular solids with internal microstructures, which enable to reduce some environmental loads because of their lightweight bodies, provide a variety of the additional mechanical properties: high-specific stiffness, shock absorption, electromagnetic shielding/transmission, thermal conductivity/insulating and so on [1]. In particular, their large deformability with a difussionless phase transformation is one of fascinated solid properties. For instance, some auxetic deformations with a negative Poisson's ratio is a kind of phase transformations in cellular solids [2]. The auxetic materials are formed by specific microstructures and the coordinated rotations of rigid squares or triangles, that are the components of the microstructure, give rise to such a mechanism [3]. Our recent work focused on the geometry and transformation of a planar microstructure to establish the relationship between the overall mechanical property and the local rotational mechanics of the connections [4, 5]. In this paper, we propose the novel structure with D_8 symmetry and cast the new concept of internal structural transformation that enables the remarkable change of its elastic stiffness.

PROPOSITION OF D8 SYMMETRY STRUCTURES

Structures

Fig. 1 illustrates the proposed structures with D_8 symmetry, which have an 8-fold rotation center and 8 axes of symmetry passing through the center. These structures are made up of identical beam segments with equal length and they have particular connectivity between the beam segments. In the structural unit as shown in Fig 1 (a), the adjacent beam segments painted in the same color, blue or red, are rigidly connected together on all the pivot joints. In other word, the structural unit is built from a pair of rigid cross-bodies and 8 rigid squares, and all of them are pivotally connected one another according to the adequate configuration. Based on the basic structural unit, the structure is extensible in a concentric fashion to large size without limitation. One example is shown in Fig. 1(b).

Kinematic motions of the proposed structure

A family of unconventional structures can be observed in some mechanical systems such as umbrellas and foldable chairs, that are designed to contain internal mobilities that allow large shape transformations to satisfy practical requirements. Such structures are called "motion structures" by Z. You [6]. A mechanism of a motion structure is a combination of rigid bodies connected by movable joints for the purpose of transmitting motion. Our proposed structure is also one of motion structures and it allow large shape transformations due to the coordinated rotations of rigid beam segments via the pivot joints [5]. Figs. 2(a) and (b) show two types of motions of the proposed structural unit. One is the transformation into square cells which are tilted at a 45° (Fig. 2(a)) and the other is the transformation into square cells parallel to the

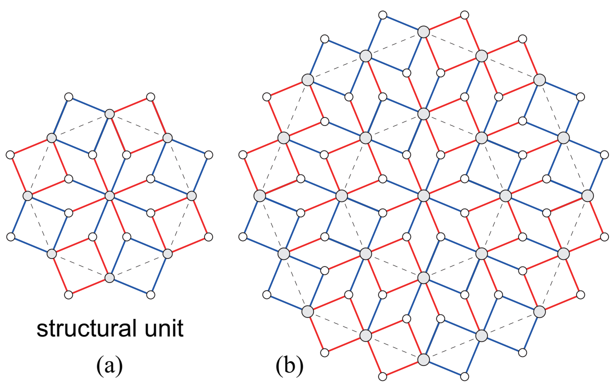

Figure 1. D_8 symmetry structures. The adjacent beam segments painted in the same color, blue or red, are rigidly connected together on all the pivot joints.

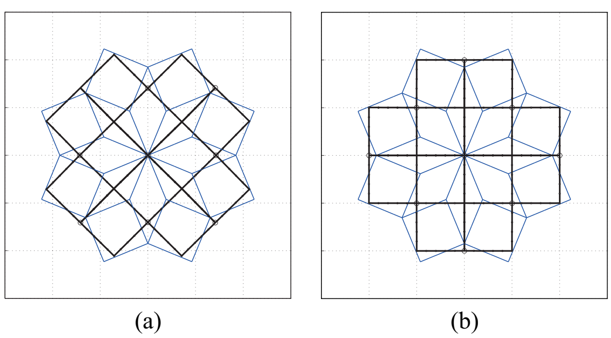

Figure 2. Two types of transformations of the proposed structure: (a) Motion I; (b) Motion II.

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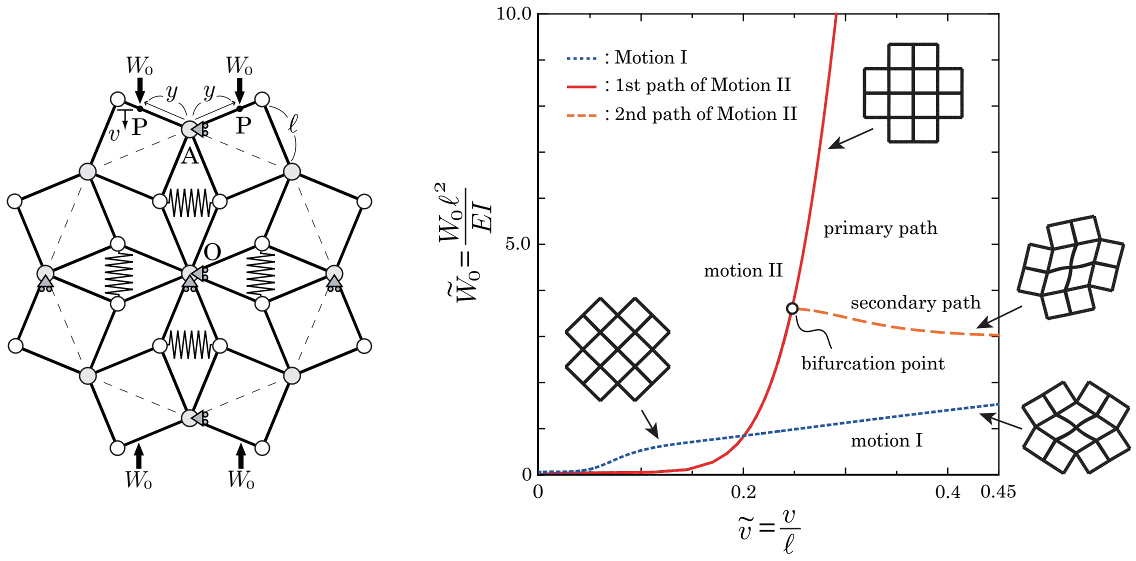

(a) (b)

Figure 3. (a) Analytical model for Motion I, that takes into account the pseudo-contact interaction between the close two segments by inserting the nonlinear springs: $f = k_1 u + k_2^{15}$, where f and u are the force and displacement of the spring. Each material parameter is that $EA = 5.0 \times 10^4$, $EI = 5.0 \times 10^3$, $\ell = 10$, $k_1 = 1.0 \times 10^{-3}$, $k_2 = 5.0 \times 10^{12}$; (b) Dimensionless load-displacement curves.

horizontal and vertical axes (Fig. 2(b)). Hereafter, the former transformation is called as Motion I and the latter as Motion II. The two motions can be characterized as the rigid rotations of 8 surrounding square cells, and each rotation of the squares in Motion I corresponds to the opposite of each rotation of the squares in Motion II. The large size structure illustrated in Fig. 1(b) also behaves the same two motions with rigid square rotations.

Mechanical properties of the proposed structure

We here discuss the mechanical properties of the D_8 symmetry structure, induced by Motion I and Motion II. Let us consider the boundary condition illustrated in Fig. 3(a). The two axisymmetric points of compressive loading W_0 are located at P, and the arc-length along surface $\overline{\mathrm{AP}}$ is described by the parameter, $\tilde{y} = y/\ell$. In the analytical model, the nonlinear springs are implemented to avoid the overlapping of beam segments during the process of transformation. In the case of Fig. 3(a), the stiffness of the nonlinear spring is infinitesimal at an initial configuration, but, under the contraction of the spring, the stiffness increases so high that the close two segments are locked like a contact when the structure transforms into square cells. In the analytical model, selecting the transformation of Motion I or Motion II depends simply on the loading point $\tilde{y}$. Based on the balance equation of moment, we can lead the unstable equilibrium point $\tilde{y}_{\mathrm{e}} = 2\tan(\pi/8)/(\sqrt{2} - \tan(\pi/8)) \approx 0.8284$. Thus, Motion I is occurred as $\tilde{y} < \tilde{y}_{\mathrm{e}}$ and Motion II as $\tilde{y} > \tilde{y}_{\mathrm{e}}$.

Fig. 3 shows two load-displacement curve, where $\tilde{v}$ is a dimensionless vertical displacement at the loading point P. One is the curve of Motion I as $\tilde{y} = 0.8$. The other is the curve of Motion II as $\tilde{y} = 1.0$, which has two paths branched via a bifurcation point. In the primary path (solid line) of Motion II, it can be observed that the stiffness increases rapidly just as transforming into square cells and the structure has a potential to reach up to the axial-stiffness of a beam segment. However, it has another potential to give rise to buckling as the secondary path (dashed line). After buckling, the applied load gradually decreases and it has negative stiffness. Fig. 3 indicates that the stiffness for Motion II may be over forty times higher than the stiffness for Motion I due to the extremely anisotropy of square cells if the buckling of square cells can be avoided by applying displacement control.

Acknowledgements

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