COMPLICATED STRESS OSCILLATIONS INDUCED IN FUNCTIONALLY GRADED MATERIAL FILMS

Fumihiro Ashida [* a)], Sei-ichiro Sakata [*], and Tatsuya Ohtsuka [**]

[*] *Department of Electronic Control Systems Engineering, Shimane University, Matsue 690-8504, Japan*

[**] *Interdisciplinary Graduate School of Science and Engineering, Shimane University, Matsue 690-8504, Japan*

Summary One-dimensional dynamic elastic and thermoelastic problems in thin functionally graded material (FGM) films subjected to an impact pressure or a thermal shock are analyzed and exact solutions are derived. Obtained numerical results reveal that stress oscillations are quasi-periodic or unsteady when the acoustic impedance depends on the position, whereas they are periodic and monotonic when the impedance is independent of the position. This effect is useful for controlling a complicated stress oscillation induced in a thin FGM film.

INTRODUCTION

Numerous papers discussed dynamic problems in various FGM solids subjected to an impact pressure or a thermal shock [1, 2]. A few papers reported that a quasi-periodic vibration was produced in an FGM structure and therefore time histories of stresses were complicated [2]. This paper investigates quasi-periodic and unsteady stress oscillations induced in thin FGM films by the elastic wave propagation. Applying the transformation of the position variable and the Laplace transform, the analytical solutions to the one-dimensional dynamic elastic and thermoelastic problems in the FGM films are respectively derived for the case where nonhomogeneous material constants are given as exponential functions of the position. Numerical results obtained from the analytical solutions disclose that the acoustic impedance governs whether the quasi-periodic stress oscillation or the unsteady thermal stress oscillation occurs in the FGM films.

PROBLEM STATEMENT AND ANALYSES

Dynamic elastic problem

It is assumed that an infinite thin FGM film depicted in Fig. 1 is initially in stress-free state, one surface is subjected to a uniform impact pressure $\sigma_0 H(t)$, and the other surface is fixed to a rigid body. The initial and boundary conditions are

$$u_z = u_{z,t} = 0 \ \text{ at } \ t=0, \ u_z = 0 \ \text{ on } \ z=0, \ \sigma_{zz} = -\sigma_0 H(t) \ \text{ on } \ z=l \tag{1}$$

where $u_z(z,t)$ is the displacement, $\sigma_{zz}(z,t)$ is the stress, t is the time, l is the thickness of the FGM film, $H(t)$ is Heaviside's unit step function, and $u_{z,t} = \partial u_z / \partial t$. The constitutive equation and the equation of motion are

$$\sigma_{zz} = \{\lambda(z) + 2\mu(z)\} u_{z,z}, \quad \sigma_{zz,z} = \rho(z) u_{z,tt} \tag{2}$$

where $\lambda(z)$ and $\mu(z)$ are the Lamé elastic constants and $\rho(z)$ is the density. They are assumed to be expressed as $\{\lambda(z), \mu(z)\} = \{\lambda_0, \mu_0\} e^{pz}$ and $\rho(z) = \rho_0 e^{qz}$. Introducing the variable transformation defined as $\zeta = e^{\delta z}$ and applying the Laplace transform, the stress is obtained when the nonhomogeneous parameters are taken to be $p = \delta$ and $q = 3\delta$:

$$\sigma_{zz} = -\sigma_0 \left\langle 1 + 2\sum_{n=1}^{\infty} \frac{[\beta_n \zeta \cos\{\beta_n(1-\zeta)\} + \sin\{\beta_n(1-\zeta)\}]\cos\{c_{e0}\,\delta\,\beta_n t\}}{\beta_n[(\zeta_l - 1)\zeta_l \beta_n \sin\{\beta_n(1-\zeta_l)\} + \cos\{\beta_n(1-\zeta_l)\}]} \right\rangle \tag{3}$$

where δ is an arbitrary constant, $\zeta_l = e^{\delta l}$, $c_{e0} = \sqrt{(\lambda_0 + 2\mu_0)/\rho_0}$, and β_n are the roots of the equation:

$$\beta_n \zeta_l \cos\{\beta_n(1-\zeta_l)\} + \sin\{\beta_n(1-\zeta_l)\} = 0 . \tag{4}$$

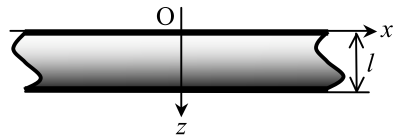

Fig. 1 An infinite thin FGM film

Dynamic thermoelastic problem

Consider an infinite thin FGM film depicted in Fig. 1, again. It is assumed that the film is initially at the reference temperature, one surface is suddenly exposed to a uniform heating temperature $T_c H(t)$, and the other surface is kept at the reference temperature. Both surfaces are taken to be stress-free. The initial and boundary conditions are given by

$$T = 0 \ \text{ and } \ u_z = u_{z,t} = 0 \ \text{ at } \ t=0, \ T = T_c H(t) \ \text{ and } \ \sigma_{zz} = 0 \ \text{ on } \ z=0, \ T=0 \ \text{ and } \ \sigma_{zz}=0 \ \text{ on } \ z=l \tag{5}$$

where $T(z,t)$ is the temperature change from the reference temperature. The governing and constitutive equations are

$$\{k(z) T_{,z}\}_{,z} = C_v(z)\rho(z) T_{,t}, \quad \sigma_{zz} = \{\lambda(z) + 2\mu(z)\} u_{z,z} - \{3\lambda(z) + 2\mu(z)\}\alpha(z) T, \quad \sigma_{zz,z} = \rho(z)\, u_{z,tt} \tag{6}$$

where $k(z)$ is the thermal conductivity, $C_v(z)$ is the specific heat, and $\alpha(z)$ is the coefficient of linear thermal expansion. In order to solve Eqs. (6) subject to the initial and boundary conditions (5), it is assumed that the material constants are given as $k(z) = k_0 e^{az}$, $\kappa(z) = (k_0 / C_{v0}\,\rho_0) e^{bz}$, $\{\lambda(z), \mu(z)\} = \{\lambda_0, \mu_0\} e^{pz}$, $\rho(z) = \rho_0 e^{qz}$, and $\alpha(z) = \alpha_0 e^{fz}$. Introducing the variable transformation defined as $\zeta = e^{\delta z}$ and applying the Laplace transform, the temperature change $T^*(\zeta, s)$, displacement $u_z^*(\zeta, s)$, and stress $\sigma_{zz}^*(\zeta, s)$ are obtained in the Laplace transform domain:

$$u_z^*(\zeta,s) = \zeta^{-\gamma}[D\{A I_{-\gamma}(\omega_1\zeta) + B K_{-\gamma}(\omega_1\zeta)\} + F\zeta\{A I_{-\gamma+1}(\omega_1\zeta) - B K_{-\gamma+1}(\omega_1\zeta)\} + \zeta^{-1}\{L I_{-\gamma}(\omega_2\zeta) + M K_{-\gamma}(\omega_2\zeta)\}] \tag{7}$$

---

a) Corresponding author. Email: ashida@ecs.shimane-u.ac.jp

where s is the Laplace parameter, $I_\gamma(\zeta)$ and $K_\gamma(\zeta)$ are the modified Bessel functions, D, F, and ω_i are known coefficients, A, B, L, and M are unknown coefficients determined from the boundary conditions, and $\gamma=-a/2\delta$. The temperature change $T^*(\zeta,s)$ and stress $\sigma^*_{zz}(\zeta,s)$ are omitted. Assuming that $a=p=\delta$, $b=-f=-2\delta$, and $q=3\delta$ and taking the inverse Laplace transform, the analytical solution for the temperature change, displacement, and stress is obtained, but omitted here because the functions are lengthy.

NUMERICAL RESULTS

The thickness of the film and the arbitrary constant are taken to be $l=10^{-8}$ m and $\delta l=0.4$. The reference material is considered to be aluminum alloy with the following properties:

$$E_0 = 68.9\times10^9\,\mathrm{N\,m^{-2}},\ \rho_0 = 2700\,\mathrm{kg\,m^{-3}},\ \nu = 0.33,$$
$$k_0 = 155\,\mathrm{W\,m^{-1}K^{-1}},\ C_{v0} = 964\,\mathrm{J\,kg^{-1}K^{-1}},\ \alpha_0 = 23.4\times10^{-6}\,\mathrm{K^{-1}}.$$

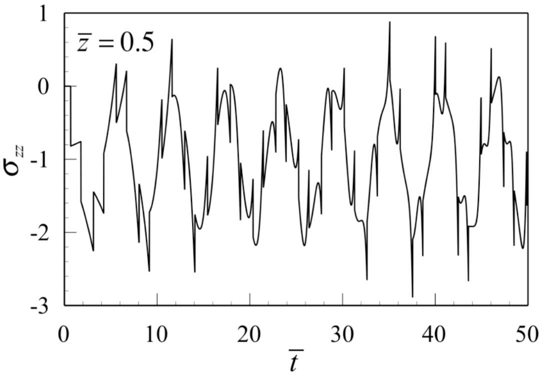

Fig. 2 Quasi-periodic stress oscillation

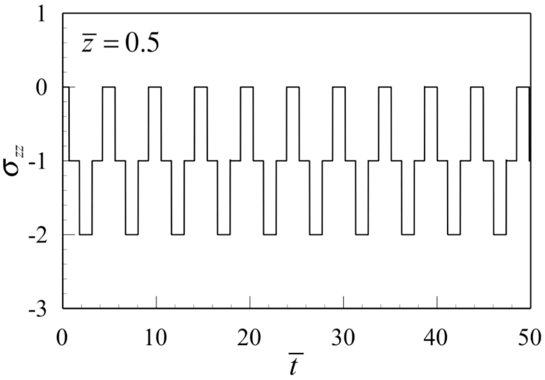

Fig. 3 Periodic stress oscillation

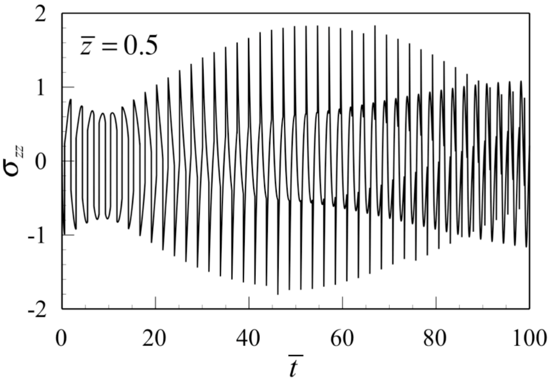

Fig. 4 Unsteady thermal stress oscillation

Dynamic elastic problem

Figure 2 illustrates the numerical results obtained from Eq. (3), where the nondimensional stress, coordinate variable, and time are represented by $\bar{\sigma}_{zz}=\sigma_{zz}/\sigma_0$, $\bar{z}=z/l$, and $\bar{t}=c_{e0}t/l$. It is seen from the figure that the stress oscillation is quasi-periodic and the waveform changes complicatedly. The reason of such behavior is due to period differences of higher-order mode components of the stress wave. It is clear from Eq. (3) that the periods are governed by the roots β_n of Eq. (4). The ratios are $\beta_2/\beta_1=3.497$, $\beta_3/\beta_1=5.885$, and $\beta_4/\beta_1=8.261$. In the case of $p=\delta$ and $q=3\delta$, the acoustic impedance defined as $Z=\sqrt{\{\lambda(z)+2\mu(z)\}\rho(z)}$ is a function of the position.
Now, the acoustic impedance becomes constant in the case of $p=-\delta$ and $q=\delta$. Then the ratios are $\beta_2/\beta_1=3$, $\beta_3/\beta_1=5$, and $\beta_4/\beta_1=7$ and therefore the stress oscillation is periodic and monotonic as shown in Fig. 3. Comparing Figs. 2 and 3, the maximum amplitude of the stress oscillation when the acoustic impedance depends on the position is about double of it when the acoustic impedance is independent of the position.

Dynamic thermoelastic problem

Figure 4 illustrates the unsteady thermal stress oscillation in the case of $p=\delta$ and $q=3\delta$, where the nondimensional thermal stress is represented by $\bar{\sigma}_{zz}=\sigma_{zz}/(3\lambda_0+2\mu_0)\alpha_0 T_c$. The oscillation amplitude is found to change as the time advances. Similar to the results for the foregoing problem, the thermal stress oscillation is periodic and monotonic in the case of the constant acoustic impedance, namely $p=-\delta$ and $q=\delta$, but the figure is omitted here. The maximum amplitude of the thermal stress oscillation when the acoustic impedance depends on the position is about double of it when the acoustic impedance is independent of the position. The same phenomenon can be observed in numerical results for a one-dimensional generalized thermoelastic problem based on Lord and Shulman's theory which are obtained by employing the method of characteristic [1].

CONCLUSIONS

In this paper, the analytical solutions to the one-dimensional dynamic elastic and thermoelastic problems in the FGM films are obtained. Numerical results demonstrate that the elastic and thermoelastic stress oscillations are quasi-periodic or unsteady when the acoustic impedance depends on the position. Such complicated stress oscillations can be suppressed when nonhomogeneous material properties are designed so that the acoustic impedance is independent of the position.

References

[1] Sumi, N. and Sugano, Y.: Thermally Induced Stress Waves in Functionally Graded Materials with Temperature-dependent Material Properties, *Journal of Thermal Stresses*, **20**: 281-294, 1997.

[2] Yas, M. H., Shakeri, M., Heshmati, M., and Mohammadi, S.: Layer-wise Finite Element Analysis of Functionally Graded Cylindrical Shell under Dynamic Load, *Journal of Mechanical Science and Technology*, **25**: 597-604, 2011.