THE RESONANT PHENOMENA IN THE ELASTIC OIL PIPELINE, SPENT TO SEA WATER AND SUBJECTED TO IMPULSE STRESSING

Anatoly Egorov, Murat Baimukhametov
Institute of Mechanics and Machine Science, Almaty, Kazakhstan

Summary Using a method of Lejbenzon-Ishlinsky research of stability of deformable systems dynamic azimuthal forms of loss of stability of the oil pipeline spent in sea water and subject pulse stressing in its vicinity in the form of earthquake or explosion are studied. The resonant phenomena in the oil pipeline, leading to its destruction are investigated.

THE BASIC PRECRITICAL STRESS-STRAIN STATE

The static stress-strain state of the oil pipeline laid in sea water is investigated. Oil (a viscous incompressible liquid) flows on the oil pipeline laminarly. The friction forces of tinting strength and normal pressure, linearly changing on length of the oil pipeline operate on an internal surface, and even distributed pressure from sea water operates on an external surface. The problem decision about meridional pipe deformation is given.
The axial w and radial u movings are functions of cylindrical coordinates r, z. The decision is given in dimensionless variables $x, \xi : x = \frac{r}{a}, \xi = \frac{z}{a}$, where a – external radius of a pipe, and

$$\frac{b}{a} = x_1 \le x \le 1, \ 0 \le \xi \le L = \frac{l}{a}. \tag{1}$$

Here b – internal radius of a pipe, l – length of run of the oil pipeline. The decision is spent with use of harmonious functions [1]:

$$\varphi_0 = 1, \varphi_1 = \xi, \varphi_2 = \frac{1}{2}(2\xi^2 - x^2), \varphi_3 = \frac{1}{2}(2\xi^3 - 3\xi x^2) \text{ and s.f.,} \tag{2}$$

$$\psi_0 = \ln x, \psi_1 = \xi \ln x, \psi_2 = \varphi_2 \ln x + \frac{1}{3}(x^2 + \xi^2) \text{ and s.f.,} \tag{3}$$

with which help moving and pressure are expressed.
Boundary data:

$$\begin{aligned} &\text{at } x = 1: \ \sigma_r = 0, \quad \tau_{rz} = 0; \\ &\text{at } x = x_1: \ \sigma_r = -q_1 \xi, \ \tau_{rz} = -\tau_1. \end{aligned} \tag{4}$$

The problem decision is expressed through ax symmetric harmonious functions:

$$\begin{aligned} b_0 &= A\varphi_3(x,\xi) + B\psi_1(x,\xi), \\ b_3 &= C\varphi_2(x,\xi) + D\psi_0(x,\xi), \end{aligned} \tag{5}$$

A, B, C, D – arbitrary constant of integration of balance equation . They are defined from boundary data (4).
Lame's problem for a hollow cylinder under the influence of uniform normal pressure upon its external surface from sea water is separately considered.
It is enough to keep only one function b_r in the solution of Papkovich-Neuber [1]:

$$b_r = C_1 x + \frac{C_2}{x}, \tag{6}$$

where C_1 and C_2 – arbitrary constant of integration. As far as for the linear theory of elasticity the superposition principle is fair, the decision of the formulated problem is obtained by summation of moving and pressure caused by moving oil in the oil pipeline and pressure of sea water.

THE OIL PIPELINE INSTABILITY. THE RESONANCE CONDITION.

Dynamic forms of the oil pipeline instability subject to impulsive loading are investigated. At the beginning, internal and external boundary value problems for free azimuthal fluctuations of the elastic oil pipeline are solved.
The equation of dynamic fluctuations of an elastic body in movements in the vector form looks like [1]:

$$\frac{1}{1-2\nu} grad\,div\vec{U}_* + \nabla^2 \vec{U}_* = \frac{\rho}{G}\ddot{\vec{U}}_*, \tag{7}$$

where operator Laplas in polar system of coordinates

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}, \tag{8}$$

$\varphi -$ azimuthal corner. Displacement vector can be represented as:

$$\vec{U}_*(t,r,\varphi) = \exp(ift)\vec{U}_{**}(r,\varphi), \tag{9}$$

$f -$ frequency.

Relative volume expansion Θ_{**} submits to the equation of Helmholtz:

$$\nabla^2\Theta_{**} + \frac{\rho}{2G}\frac{1-2\nu}{1-\nu} f^2\Theta_{**} = 0, \tag{10}$$

where

$$\Theta_{**} = \frac{\partial u_{**}}{\partial r} + \frac{u_{**}}{r} + \frac{1}{r}\frac{\partial v_{**}}{\partial \varphi}, \tag{11}$$

$u_{**}, v_{**} -$ vector components $\vec{U}_{**}$.

By division of variables and the subsequent decision of system of the ordinary differential equations, decisions for internal and external regional problems for a pipe are found.

The secular equation for detection of frequencies of free fluctuations of a pipe is received. According to a method of Lejbenzon–Ishlinsky [2,3] boundary stability data of azimuthal fluctuations of the oil pipeline laid in sea water are deduced. As a result of the oil pipeline instability originally cylindrical boundary surfaces take the form, different from cylindrical, owing to turns of boundary elements. Using in the found boundary stability data components of the basic precritical condition, and also components of disturbance, system of the linear homogeneous algebraic equations with the unknown quantity - arbitrary constants can be received, from a condition of nontrivial decisions it is found the secular equation for detection of bifurcation frequencies of the constrained dynamic fluctuations of the oil pipeline. We deduce a measure of a dynamic susceptibility of system to constrained bifurcation forces [4]:

$$\lambda = \frac{c^2 + f^2}{\sqrt{(c^2 - f_1^2 + f^2)^2 + 4c^2 f_1^2}}, \tag{12}$$

where $f -$ the real frequency of free fluctuations, $f_1 -$ real, c – imaginary parts of complex frequency $(f_1 - ci)$ bifurcation the constrained fluctuations, where $c > 0$, i – imaginary unit. At $f_1 \to f$ this measure sharply accrues. Meanwhile the resonance condition occurs which can lead to oil pipeline destruction.

CONCLUSIONS

The problem about meridional pipe deformation is solved. By means of the deduced boundary stability conditions, the secular equation for definition of frequencies constrained bifurcation fluctuations of the oil pipeline, comparable with frequency of free fluctuations is received. The measure of a dynamic susceptibility of the oil pipeline for constraining bifurcation forces is defined. According to this measure, it may be judged about the resonant phenomena in the oil pipeline, leading to its destruction.

References

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