EFFECTS OF FINITE DEFORMATION IN THE DEFLECTION OF TRANSVERSELY-ISOTROPIC SEMILINEAR ELASTIC THIN PLATE

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Summary This work examined the deflection of a transversely-isotropic elastic thin plate in finite deformation. Using semilinear material it was observed that harmonic forces exist within the planes of the plate whereas these forces vanished in the Sophie-Germain model for small deformation. Also, compared to the case of small deformation, we observed higher flexural rigidity and vibration frequencies; while deflections of both dielectric and conducting elastic thin plates, loaded by electromagnetic forces, were relaxed by the in-plane harmonic forces.

INTRODUCTION

Sophie-Germain equations with accompanying pertinent boundary conditions provide the first classical basic relations for investigating the deflection of elastic thin plate in small deformation theory [1]. Now, the question is posed, *what would be the equivalent to these equations in the case of finite deformation theory, using semilinear material?* The obligation or indeed burden of the need to adopt finite deformation approach in problems of solid/continuum mechanics is imposed by the fact that the small deformation theory is a gross approximation of the process being modelled [2] with the consequent implication that some effects (at times vital ones at that) are inadvertently inhibited [3]. Infact, an elastic material under simple shears in small deformation approach exhibits only shear stresses whereas in finite deformation approach the body exhibits additionally normal stresses, in what is called *ponyting effect*. Similarly, a bar in torsion under finite deformation exhibits *Kelvin effect*; the presence of axial stresses, which small deformation failed to detect. Furthermore, finite deformation procedure helps us to detect 'grip effect' in a tube torsioned from the interior (i.e. internal radius of a torsioned tube decreases), in what is referred to as '*hold effect*' [3]. However, adopting the finite deformation approach provokes collateral challenge(s). One of the principal challenges posed is on how to obtain suitable constitutive relation and tractable equation of state. Here, we consider the deflection of a transversely-isotropic elastic thin plate in the context of finite deformation approach and deduce an equivalent to the Sophie-Germain equation for small deformation. For this, we highlight pertinent material energy function (semilinear material) [3] and invoke the concept of hyperelasticity. We take the tensor Frechet derivative with respect to the energy conjugate geometry of deformation which is the gradient tensor of the position vector in the current configuration. This enables us obtain constitutive equation and consequently deduce the equation of state. A number of equilibrium and dynamic problems are considered for the transversely-isotropic elastic thin plate: the vibration of this plate is investigated and the frequencies of vibration for varying values of Poisson ratio are obtained and compared to the case of linear elasticity. Furthermore, the special case when the plate is under the influence of electromagnetic forces [4,5] is also considered.

DEFLECTION MODEL FOR TRASVERSELY-ISOTROPIC THIN PLATE

We recall the Sophie-Germain equilibrium equation for obtaining the deflection w of elastic thin plate under small deformation

$$D\nabla^4 w = p^*;\ \ \nabla^4 \equiv (\nabla^2)(\nabla^2) = \left(\partial^4/\partial x_1^4 + 2\partial^4/\partial x_1^2\partial x_2^2 + \partial^4/\partial x_2^4\right), D = Eh^3/12(1-\upsilon^2), \qquad (1)$$

where D is the plate stiffness, p^* is the load, h is the plate thickness and υ is the poisson's ratio [1].
In the case of finite deformation, let the elastic thin plate Ω, be a subset of three dimensional Euclidean space E^3 (i.e $\Omega \subset E^3$). We take its geometry of transformation from initial configuration $\Omega_0(\mathbf{r})$ with position vector $\mathbf{r} = X_1\mathbf{k}_1 + X_2\mathbf{k}_2 + X_3\mathbf{k}_3$ into a current configuration $\Omega(\mathbf{R})$ with position vector $\mathbf{R} = x_1\mathbf{k}_1 + x_2\mathbf{k}_2 + x_3\mathbf{k}_3$ as the gradient tensor in the initial configuration $\overset{0}{\nabla}\mathbf{R} \equiv \mathbf{r}^1\mathbf{R}_1 + \mathbf{r}^2\mathbf{R}_2 + \mathbf{r}^3\mathbf{R}_3 = \widetilde{U}\cdot\widetilde{O}^H$,

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where $\mathbf{r}_m$, $\mathbf{r}^m$ are the respective covariant, contravariant base vectors in the initial configuration Ω_0 and $\mathbf{R}_m$ are the covariant base vectors in the current configuration Ω, $\mathbf{k}_1$ are orthonormal base vectors, $m = 1, 2, 3$, while the gradient tensor has been decomposed into strecth symmetric tensor $\widetilde{U}$ and orthogonal rotation tensor $\widetilde{O}^H$. Now, using semilinear material, the energy function in the case of isotropy and transversely-isotropy respectively is

$$W = \mu S_2 + 1/2\lambda S_1^2;\ \ W = \lambda_2 S_2 + 1/2\lambda_1 S_1^2 + \lambda_0 S_0, \tag{2}$$

where S_1 and S_2 are invariants of the deformation geometry, $S_1 = \widetilde{\mathbf{E}} \cdot \cdot (\widetilde{\mathbf{U}} - \widetilde{\mathbf{E}}) \equiv I_1(\widetilde{\mathbf{U}} - \widetilde{\mathbf{E}})$, $S_2 = I_1(\widetilde{\mathbf{U}} - \widetilde{\mathbf{E}})^2$. λ and μ are the lame constants. $S_0 = c \cdot \widetilde{\mathbf{U}}^2 \cdot c$ is an additional invariant of deformation, due to anisotropy. $\mathbf{c}$ is the unit vector characterizing the direction of anisotropy. $\lambda_1, \lambda_2, \lambda_0$ are the effective material characteristics that result from applying homogenization theory to composite/heterogeneous media that in turn has reduced to a transversely-isotropic one. In the case of randomly unidirectional fibre reinforced composites or a laminar layered media the material constants are the effective moduli [3]. Now, invoking the hypothesis of hyperelasticity of Cauchy-Truesdell [3], we take the frechet derivative of the energy function (2) with respect to the geometry of the deformation (deformation gradient) $\overset{0}{\nabla} \mathbf{R}$ and obtain the constitutive law, Piola stress tensor $\widetilde{\mathbf{P}}$ to which it is energy conjugate:

$$\widetilde{\mathbf{P}} \equiv \partial W / \partial \overset{0}{\nabla} \mathbf{R} = 2\lambda_2 \overset{0}{\nabla} \mathbf{R} + (\lambda_1 S_1 - 2\lambda_2)\widetilde{\mathbf{O}}^H + \lambda_0 \mathbf{cc} \cdot \overset{0}{\nabla} \mathbf{R}.$$

We now reflect the geometry of deformation in it to obtain the interested model

$$D\nabla^4 w + D_* \nabla^2 w = p^*, \tag{3}$$

where $D \equiv (2\lambda_2 + \lambda_1)h^3/12$ is now the stiffness of the thin plate and $D_* \equiv (2\lambda_2 + \lambda_1)h$ is the coefficient of the new term that emerged naturally, purely from derivation, due to finite deformation consideration; it constitutes the amplitude of the damping force in the plane of the plate.

CONCLUSION

In deriving equation (3), by comparison to equation (1) which is the Sophie-Germain equation for deflection of isotropic elastic thin plate in small deformation, an additional term emerged. This new term is an effect of finite deformation consideration: it constitutes an harmonic term which has a damping effect on the plate. The new effect and implications on the various problems considered are discussed in the full text. Further, the study showed that finite deformation approach to elasticity problems provides ample opportunity to navigate media and capture important effects that small deformation theory might have failed to apprehend.

References

[1] Amenzade, Y. (1979). *Theory of Elasticity.* MIR Publisher.

[2] Green, A.E. and Adkins, J. (1960). *Large Elastic Deformations.* Oxford University Press.

[3] Akinola, A., 1999. *On interacting longitudinal-shear waves in large elastic deformation of a composite cylindrical laminate.* I.J. Nonlinear Mechanics, *(34)*, 405-414.

[4] Haym, B. (1998). *Mechanical Vibration: Analysis, Uncertainties and Control.* Prentice Hall, New Jersey.

[5] Ferraro, V. C. A., 1962. *Electromagnetic Theory.* The Athlone Press, Univ. of London.