THEORY OF FERROMAGNETIC MICROPOLAR ELASTIC THIN SHELLS, PLATES AND BARS

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Summary This paper presents the theory of ferromagnetic micropolar elastic thin bodies. We start from the three-dimensional boundary-value problem of micropolar theory of elasticity for ferromagnetic body. Then, a set of hypotheses is formulated, based on which the general applied theories of statics and dynamics of thin shells, plates and bars are constructed.

INTRODUCTION

The problem of construction of the theory of elasticity, when massive and massive pairs of forces are taken into consideration, relates to the study of stress-strain state for materials with ferromagnetic properties [1, 2]. It is important, therefore, to construct the theory of ferromagnetic micropolar elastic thin bodies. In this paper, on the basis of three-dimensional boundary-value problem of micropolar theory of elasticity for ferromagnetic body, the general applied theories of statics and dynamics of thin shells, plates and bars are constructed.

EQUATIONS OF MAGNETOELASTICITY

We assume that ferromagnetic micropolar elastic shell of constant thickness $2h$ is in the given initial magnetic field. The basic system of equations of magnetoelasticity consists of three groups of systems of interrelated equations:
1) The system of equations of asymmetric theory of elasticity with consideration of massive forces and moments of electromagnetic origin and boundary conditions on the frontal surface and on the surface of the shell edge (where Maxwell tensor of stresses of magnetic origin is taken into account).

$$M_1\left(\Phi,\phi^{\text{int}}\right)=0, \qquad M_1^f\left(\Phi,\phi^{\text{int}}\right)=M_0^f, \qquad M_1^b\left(\Phi,\phi^{\text{int}}\right)=M_0^b.$$

Here Φ,ϕ^{int} are unknown quantities of mechanical and magnetic nature respectively.

2) Equations of quasi-stationary electrodynamics for the shell, which are symbolically written as:

$$M_2\left(\phi^{\text{int}}\right)=0$$

3) Equations of quasi-stationary electrodynamics (written in symbolically form) in the field of infinite vacuum space surrounding the shell:

$$M_3\left(\phi^{ext}\right)=0$$

Here ϕ^{ext} are unknown quantities of magnetic origin in the field of infinite space surrounding the shell. Continuity conditions on the surface of the shell and conditions in infinity should be added to the previous two systems of equations of the electrodynamic part of the problem:

$$N\left(\phi^{\text{int}},\phi^{ext}\right)=0, \quad \mathrm{P}^{\infty}\left(\phi^{ext}\right)=0.$$

METHOD OF HYPOTHESES

It is assumed that the shell is thin and on the basis of this fact the problem of reduction of three-dimensional problem of ferromagnetic micropolar magnetoelectricity to general applied two-dimensional boundary-value problem of the theory of shells is considered. To solve this problem, we shall apply the method of hypotheses [3]. The hypotheses are formulated on the basis of qualitative results of the asymptotic method of integration of the above mentioned problem in thin domain of the shell. These hypotheses are of rather general character and are formulated as follows:
1) Normal to the middle surface displacement and tangential free rotations are constant functions along the shell thickness, and the tangential displacements and normal free rotation are changed by a linear law:

$$V_i=u_i(\alpha_1,\alpha_2)+\alpha_3\psi_i(\alpha_1,\alpha_2), \quad V_3=w(\alpha_1,\alpha_2), \quad \omega_i=\omega_i(\alpha_1,\alpha_2), \quad \omega_3=\Omega_3(\alpha_1,\alpha_2)+\alpha_3\iota(\alpha_1,\alpha_2),$$

where $\alpha_k\ (k=1,2,3)$ are curvilinear orthogonal coordinates accepted in the theory of shells;

2) Force stress σ_{33} can be neglected compared to the force stresses σ_{ii} in the generalized Hook's law;

3) Calculation of the deformations, bending-torsions, force and moment stresses are conducted in two stages. Firstly, for the force stresses σ_{3i} and moment stress μ_{33} we take:

$$\sigma_{3i} = \overset{0}{\sigma}_{3i}(\alpha_1, \alpha_2), \qquad \mu_{33} = \overset{0}{\mu}_{33}(\alpha_1, \alpha_2)$$

Then, the values of σ_{3i} and μ_{33} are finalized by the addition of terms, obtained by integration of corresponding equilibrium equations of (1)with the requirement, that quantities, averaged along the shell's thickness, are equal to zero;

4) Quantities α_3 / R_i can be neglected;

5) We accept that components of vector $\vec{h}$ of the magnetic field, excited in the shell, are changed by linear low along the thickness of the shell:

$$h_k = h_k(\alpha_1, \alpha_2) + \alpha_k h_k(\alpha_1, \alpha_2) \quad (k = 1,2,3)$$

(it should be noted that components of vectors of magnetic induction and magnetization of magnetic field are also changed by linear law along the thickness of the shell);

6) In determination of magnetic field in the infinite space, surrounding the body, the region of the thin shell can be replaced by its middle surface, along which the surface current with components $\left(\left[h_1^{(e)}\right], \left[h_2^{(e)}\right], 0\right)$ and the surface magnetic load with components $\left[h_3^{(e)}\right]$ will flow. Here the symbol [.] denotes the discontinuity jump of the quantities on the sides of the mentioned section.

On the basis of the accepted hypotheses, the general applied two-dimensional theory of micropolar ferromagnetic elastic thin shells is constructed (theories of plates and bars are obtained as particular cases of the theory of shells). The basic equations and boundary conditions can be introduced as follows:

$$L_1(\psi, \chi) = L_{10}, \quad L_2(\psi, \chi) = L_{20}, \quad L^b(\psi, \chi) = L_0^b .$$

Here the first equation stands for the two-dimensional system of equations of the mechanical part of the problem with consideration of forces and moments of magnetic origin; the second equation denotes the two-dimensional system of integral-differential equations of electrodynamic part of the problem in the region of the middle surface of the shell; finally, the third equation represents the boundary conditions on the contour of the middle surface of the shell; ψ and χ are unknown quantities of mechanical and magnetic origin.

CONCLUSIONS

On the basis of the constructed models of ferromagnetic micropolar elastic thin shells, plates and bars concrete applied problems of determination of stress-strain state, free and forced vibrations in bars, rectangular and circular plates, cylindrical shells are solved in case of different boundary conditions. With the help of numerical analysis of studied problems features of micropolar and magnetic properties of materials of mentioned thin bodies are revealed.

References

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