STABLE TRIVIAL CONFIGURATION OF ELASTIC SYSTEMS AFTER BIFURCATION

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Summary The possibility to design innovative elastic systems with a stable trivial path automatically recovered after bifurcation is considered. Through explicit integration of the elastica and the development of the second-order variation of the total potential energy, we show an example of elastic structure where: 1) increasing monotonic axial load, buckling can be followed by restabilization and 2) the reduction of the stiffness of one of its components can increase buckling load and reduce restabilization load up to the limit case where no bifurcation occurs.

Assuming an increasing monotonic axial load, can a structure turn automatically back to a trivial stable equilibrium configuration after a buckling load?
Thinking to classical stable and unstable bifurcations [Fig. 1 (a), (b) respectively], where the trivial path is unstable for any load greater than the critical one, we would be inclined to give a negative answer to the question above. As opposed, we provide an example of elastic structure which after a first critical (buckling) load (P^A_{cr}) there is a second critical (restabilizing) load ($P^B_{cr} > P^A_{cr}$), where the bifurcated path rejoins the trivial path [Fig. 1 (c)].

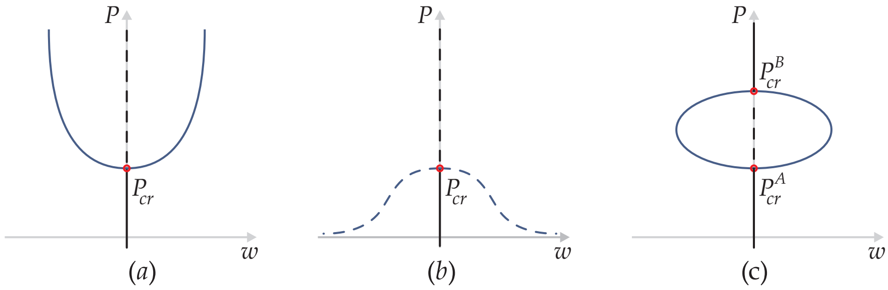

Figure 1. Bifurcated paths for structures, applied load P versus displacement w. Classical symmetric stable (a) and unstable (b) bifurcations with buckling load P_{cr}. Innovative symmetric (c) bifurcation with buckling load P^A_{cr} and restabilization load P^B_{cr}. Stable (unstable) paths are reported with continuous (dashed) line.

A further interesting feature is related to another non-intuitive concept: reduction of the stiffness of a component of the elastic system can increase buckling load and, on the other hand, can reduce restabilization load up to the limit case $k = k^*$ [Fig. 2]. Indeed for 'very soft systems' ($k < k^*$) the considered structure does not show any bifurcation at all.
Analytical solution of elastica and experimental tests are presented to confirm this innovative behaviour.

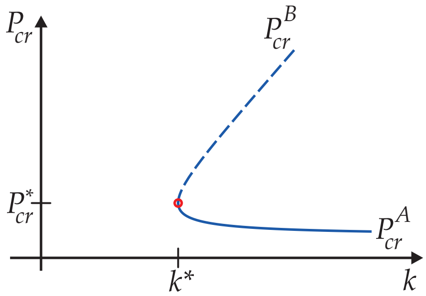

Figure 2. Bifurcation loads P_{cr} as a function of stiffness k of a component of the system. Buckling load P^A_{cr} and restabilization load P^B_{cr} are reported with continuous and dashed line, respectively. Note that decreasing the stiffness parameter k the buckling load increases while the restabilization load decreases, except for 'very soft systems' ($k < k^*$) where no bifurcation load exist.

The presented example complements previous results by Bigoni and Noselli [1], Feodosyev [2], Tarnai [3], [4], [5], Zaccaria et al. [6].

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