Mechanics of liquid crystals and quasicrystals

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Liquid crystals, in macroscopic sense, are anisotropic fluids. Therefore they belong to an intermediate phase between fluid and crystal. The mechanical behaviour of liquid crystals presents the character of both fluid and solid.

There are various types of liquid crystals, here we discuss the mechanics only for nematic, smectic and columnar liquid crystals.

Different from the behaviour of both fluid and solid, the deformation and motion of liquid crystals should be introduced a vector named director $\mathbf{n}$ apart from displacement vector $\mathbf{u}$ or velocity vector $\mathbf{V}$. In addition, the constitutive equation of liquid crystals is different from either generalized Newton's equation or generalized Hooke's equation.

For smectic and columnar liquid crystals the director in field variables and field equations can be expressed by the displacements, the discussion is easier understanding from the angles of the classical continuum mechanics.

Quasicrystals belong to another fascinating phase of condensed matter, first observed in 1982[2]. Macroscopically their main feature is that there are two different displacement fields, one is the phonon field $\mathbf{u}$, according to the terminology of physics, which is similar to the displacement field in the classical elasticity under the long-wave length approximation, the other is the phason field $\mathbf{W}$ which is new concept out of the regime of the classical continuum mechanics. The introducing of the phason field results in a great challenge to the traditional continuum mechanics. This leads to two different strain tensors, one is the Cauchy strain tensor, another is the phason strain tensor. The constitutive equations are quite different from those of claccical elasticity. The equations of motion of quasicrystals are coupled by equations of wave propagation and diffusion, are quite different from those in the classical elasto-dynamics.

The mechanics of liquid crystals are studied by de Gennes et al [4], Oswald et al [5] etc, the main attention of theirs was paid to discuss the dislocation and disclination problems, but there are some questions of the classical solutions which may be paradoxes, this suggests that the mechanics of liquid crystals needs to develop further. Brostow et al [5] started to study the crack problems in polymer liquid crystals. Because liquid crystals including nematic, smectic and columnar ones, they can be classified as monomer liquid crystals (MLCs) irrespectively of the fact whether they can or cannot polymerize into polymer liquid crystals (PLCs). That classification is due to Samuski [7] and has been used by a number of authors [8, 9]. The present model applies to MLCs and nematic, smectic and columnar phase LCs in particular. For studying crack problems of liquid crystals one must develop mechanics of liquid crystals, including three-dimensional elasticity, plasticity and dynamics.

The problem of screw dislocation in smectic A is a longstanding puzzle, de Gennes [4], Kleman [10], Pershan [11] and Landau and Lifscitz [12] presented the solution, but which may be of mistake. Pleiner [13] pointed out the problem of the solution, but the mistake could not be corrected. Fan and Li [14] develop the elasticity of smectics and find the correct solution, which modifies the mistake of the classical solution of de Gennes-Kleman-Pershan.

To develop the work on crack in liquid crystals, the plastic analysis is needed. A solution of plastic crack in one of smenctic A is found by Fan [15] based on the dislocation pile-up concept, the size of plastic zone around crack tip and crack tip opening displacement are determined.

Other solutions for plastic crack in smectic A are obtained by Fan [16]. The three-dimensional elasticity of smectic B liquid crystals is studied by Fan [17], the governing equations are reduced to three generalized harmonic equations, and an elliptic disc-shaped crack problem is solved, an approximate analytic solution is constructed. Fan and Chen [18] studied the solution of plastic crack of columnar liquid crystals, but the result is more complicated than those of smectic A liquid crystals.

Due to the space limitation, we don't list other results on mechanics of liquid crystals.

The mechanics of quasicrystals is developed by many scientists, in which the elasticity of the material is advanced for example, refer to Ding et al [19], Fan and Mai [20], Fan [21]. The systematical mathematical thoery of elasticity of quasicrystals is developed by Fan's group [21]. The elasticity, plasticity and dynamics of icosahedral quasicrystals, the most important class of the material, are well studied in the work. By introducing displacement potential, the plane elasticity of icosahedral quaiscrystals is reduced to solve the sextuple harmonic equation [22]

$$\nabla^2\nabla^2\nabla^2\nabla^2\nabla^2\nabla^2 F(x,y)=0 \qquad (1)$$

From the equation we obtain many analytic solutions of dislocations and cracks in icosahedral quasicrystals.

The complex analysis for solving the boundary value problem of equation (1) is developed, see e.g. Fan et al [23]. In terms of this method many complicated boundary value problems can be solved, the exact analytic solutions are available [21].

Fan and co-workers have paid effort to solve dynamic and plastic problems of quasicrystals, a number of solutions are constructed.

The plastic analysis of quasicrystals presents fundamental difficulty due to lack of theory of plasticity of the material. There is no any plastic constitutive equation for quasicrystals so far. We use some simplified physical models and obtain some meaningful results, for example, the size of plastic zone around crack tip and crack tip opening displacement can be determined, and comparison of results between those of quasicrystals and crystals is also given.

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