DISLOCATIONS AND INCLUSIONS IN PRESTRESSED ELASTIC MATERIALS

Davide Bigoni[*a)], Luca Argani[*] & Gennady Mishuris[**]
[*]*Department of Mechanical and Structural Engineering, University of Trento, via Mesiano 77, Trento, Italy*
[**]*Institute of Mathematical and Physical Sciences, University of Wales, Aberystwyth, UK*

Summary The effect of prestress on dislocation (and inclusion) fields in nonlinear elastic solids is analyzed by extending previous solutions by Eshelby and Willis. With a constitutive model tailored to describe the behaviour of ductile metals (the J_2-deformation theory of plasticity), we show that when the level of prestress is high enough that shear band formation is approached, strongly localized strain patterns emerge. These may explain cascade activation of dislocation clustering along slip band directions.

INTRODUCTION

The theory of dislocations (and inclusions) in solids has been thoroughly developed for elastic materials, unloaded in their natural state. We extend this theory to cover the possibility that the material be prestressed, through a generalization of solutions found by Eshelby [3] and Willis [6], by introducing an incremental formulation for incompressible materials in which the stress is related to the incremental displacement gradient [5]. As Fig. 1 shows, anisotropy strongly affects near dislocation stress fields, so that, since prestress induces an important orthotropy effect, it is expected to severely affect the material behaviour. We show that near the elliptic boundary, our solution for edge dislocations (but also, more in general, for inclusions) reveals features of the behaviour of severely deformed metals near the shear band formation threshold. In particular, we find a strong focussing of incremental fields along the directions corresponding to the shear bands (formally excluded within the elliptic region). This may induce a cascade of dislocation clustering, which may explain the fact that the amount of slip which takes places on an active shear band is three orders of magnitude greater than could be produced by the passage of a single dislocation (Cottrell, 1953).

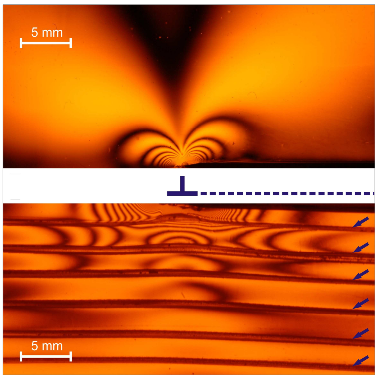

Fig.1 Photoelasticity (in monochromatic light) discloses the stress field (in-plane principal stress difference) around an edge dislocation (as sketched) in an isotropic material (upper part) and in an orthotropic material (lower part, with orthotropy axes aligned parallel and orthogonal to the dislocation line). Compared to the isotropic case (upper part), the fields become strongly elongated along the orthotropy axis parallel to the dislocation for anisotropy (lower part).

THE INCLUSION AND DISLOCATION PROBLEM

We follow and generalize Eshelby [3] and Willis [6] by considering an infinite elastic plane, homogeneouly prestressed and incompressible, containing an inclusion of arbitrary shape, also governed by the same constitutive equations, but possibly with different incremental stiffnesses. The inclusion is subject to a prescribed uniform incremental displacement gradient, that can be thought as an inelastic (for instance plastic or thermal) deformation. With a generalization of Willis, we can find expressions for the incremental displacement and mean stress fields, which are suitable for the dislocation problem. These expressions have been employed with the Green's function obtained by Bigoni and Capuani [1] to plot the results presented in Fig. 2, where maps of the modulus of incremental displacement are reported, together with the geometry of the considered edge dislocation dipole, for a J_2-deformation theory material

[a)] Corresponding author. Email: bigoni@ing.unitn.it.

[4]. The results reported in the upper part on the right of the figure pertain to a horizontal dislocation dipole placed in an infinite elastic isotropic incompressible material in the absence of prestress. Materials presetressed near ellipticity loss (shear band formation) are explored in the lower part of Fig. 2 for a horizontal dipole and a dipole inclined at 30°.

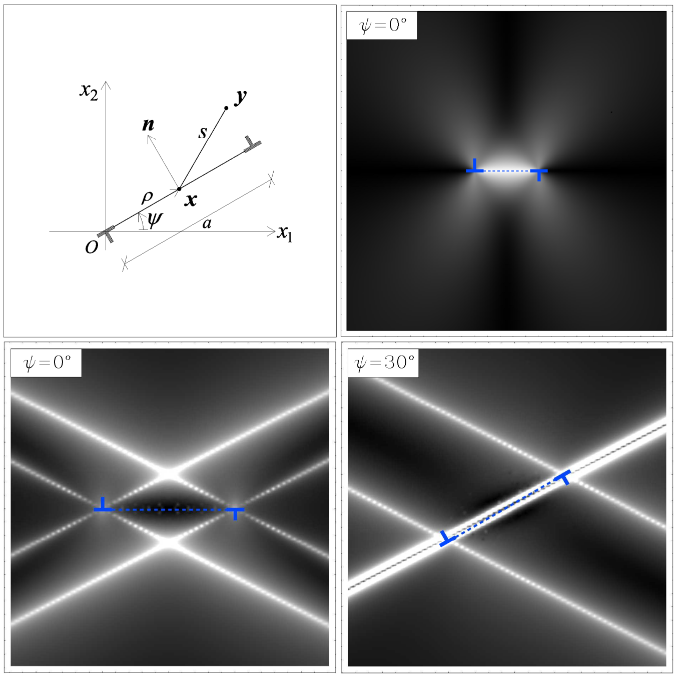

Fig. 2 Behaviour of an edge dislocation dipole in a nonlinear elastic material. Reference system (left), unloaded linear elastic isotropic case (upper part on the right), prestressed elastic material near loss of ellipticity (lower part).

CONCLUSIONS

Prestress is shown to be an important factor in the mechanics of dislocation clustering, near shear band formation in severely deformed ductile metals. A new solution for an edge dislocation, valid for incremental nonlinear elasticity, shows in fact emergence of highly localized deformation patterns, when the material is deformed near the boundary of ellipticity loss, which may trigger 'cascade' dislocation activation along shear band directions.

References

[1] Bigoni, D., Capuani, D.: Green's function for incremental nonlinear elasticity: shear bands and boundary integral formulation, J. Mech. Phys. Solids, **50**, 471-500, 2002.

[2] Cottrell, A.H.: Dislocations and plastic flow in crystals, Oxford University Press 1953.

[3] Eshelby, J.D.: The Determination of the Elastic Field of an Ellipsoidal Inclusion, and Related Problems, Proc. R. Soc. London, Ser. A, Mathematical and Physical Sciences, **241**, 376-396, 1957.

[4] Hutchinson, J.W., Neale, K. W.: Finite strain J_2-deformation theory. In: Carlson, D. E., Shield, R.T. (eds), Proc. IUTAM Symp. on Finite Elasticity, Martinus Nijhoff, The Hague-Boston-London, 237-247, 1979.

[5] Ogden, R.W.: Non-linear elastic deformations, Ellis Horwood, Chichester 1984.

[6] Willis, J. R.: Dislocations and inclusions, Pergamon Press Ltd., J. Mech. Phys. Solids, **13**, 377-395, 1965.