ANALYTICAL RELATIONS FOR RANDOM FATIGUE OF SINGLE DAMAGE VARIABLE MODELS

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Summary The Single Damage Variable (SDV) assumption characterizes many fatigue models. Their damage evolution can be described by a first-order differential equation. Is this study, some analytical relations which hold for this group of models are obtained from the general SDV constitutive equation. It is shown that data of damage evolutions under constant amplitude loads is sufficient for predicting fatigue life under any load history. Alternatively, two-level fatigue envelopes are also sufficient for life prediction. On the basis of these envelopes, an explicit integral formula for fatigue life under random loads is developed. The analytical predictions are validated by comparing with numerical simulations for two well-known SDV models under a specific simple random load. In addition, it is proven that the Palmgren-Miner rule constitutes an upper bound for random fatigue life predictions, and that two-level fatigue envelopes are anti-symmetrical.

INTRODUCTION

Fatigue failure under varying load conditions and especially random loads is prevalent in engineering applications. Nevertheless, the prediction of fatigue life in design still relies on phenomenological models. The concept of damage accumulation is used to describe material degradation during fatigue life, where damage variables are associated with the material state at a point in time. For simplicity, many models consider a Single Damage Variable (SDV) D. These models have been developed continuously from different viewpoints: phenomenological [1], continuum [2], and micromechanical [3]. SDV models are diverse and follow a wide span of manifestations of the actual damage such as stiffness degradation, ductility exhaustion, electric resistance, surface roughness and crack length.

SDV EQUATION AND BASIC RESULTS

All SDV models are based on a scalar damage variable D, whose increment at the n^{th} cycle depends on $D(n)$ and the stress amplitude $\sigma(n)$ only. It is also assumed that the damage increment does not explicitly depend on n, Stress levels are above the endurance limit, and there are no healing effects. Under these assumptions, the general SDV evolution equation is autonomous and takes the form,

$$\frac{dD}{dn} = G\left(\sigma(n), D(n)\right) \geq 0. \tag{1}$$

Material parameters are included in $G(\cdot,\cdot)$, and unlike D their number is not limited. Without loss of generality, the damage at failure is taken as $D_f = 1$. For a constant stress, (1) is separable, thus the stress life curve is found from

$$N(\sigma) = \int_0^{D_f=1} 1/G\big(\sigma, D\big)dD. \tag{2}$$

Two-level loading protocols are typically used to characterize damage accumulation. A specimen is loaded for n_1 cycles under σ_1 followed by n_2 cycles under σ_2 till failure. Under the SDV assumption, these tests give a full characterization of the damage accumulation, i.e., they can be used to predict fatigue life under a general loading protocol. Integration of (1) yields fatigue life under two-level loading,

$$\begin{cases} n_1 = \int_0^{D(n_1)} 1/G\big(\sigma_1, D\big)dD = N_1 - \int_{D(n_1)}^1 1/G\big(\sigma_1, D\big)dD \\ n_2 = \int_{D(n_1)}^1 1/G\big(\sigma_2, D\big)dD = N_2 - \int_0^{D(n_1)} 1/G\big(\sigma_2, D\big)dD \end{cases} \tag{3}$$

From (3) it can be shown that two-level high to low and low to high envelopes are anti-symmetrical, and that two-level envelopes are confined to the range $0 \leq n_1 \leq N(\sigma_1)$; $0 \leq n_2 \leq N(\sigma_2)$, where $N(\sigma)$ is the fatigue life under a constant amplitude σ.

RANDOM FATIGUE

A new analytical prediction for fatigue life under random loads as a functional of the two-level envelopes for all SDV models is developed. We assume that fatigue life is long enough with respect to the length of a representative part of the random loading. This allows averaging of (1) over different realizations. Using (1)-(3) results in the following explicit expression:

$$N\{\sigma\} = -\int_0^1 \frac{d\tilde{n}_{\text{ref}}}{\int_0^\infty \left[\frac{d\tilde{n}_{\text{ref}}}{d\tilde{n}_1}\right]_{\substack{\tilde{n}_{\text{ref}} \\ \sigma_{\text{ref}},\sigma_1}} \frac{f(\sigma_1)}{N(\sigma_1)} d\sigma_1}, \tag{4}$$

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where $f(\sigma)$ is the stress distribution function and $\tilde{n_i} = n_i/N(\sigma_i)$.
The prediction is validated using two well-known models by Hashin [1] and Manson [4]. Fatigue life under a simple random load composed of a fraction of cycles ϕ of low stress σ_L and a fraction $1-\phi$ of high stress σ_H is calculated analytically using (4). Figure 1 presents the analytical results with comparison to cycle by cycle numerical calculations and validates (4). Notice that $\phi = 0, 1$ represent constant amplitude loads, thus the corresponding fatigue lives follow Miner's rule.

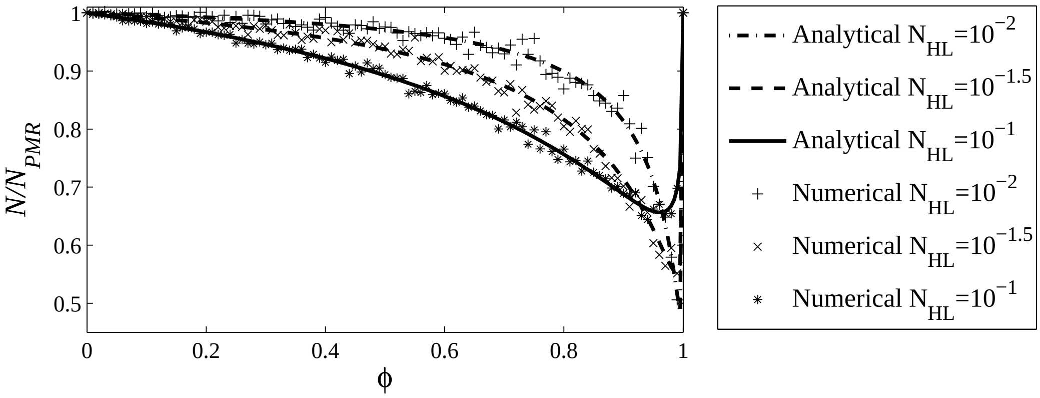

Figure 1. Analytical and numerical fatigue life predictions for Hashin's model vs. ϕ for different $N_{HL} \equiv N(\sigma_H)/N(\sigma_L)$ values; $N(\sigma_L)/N(\sigma_{\text{endurance}}) = 0.5$. Numerical results (cycle-by-cycle calculation of residual life) are for a white noise loading, normalized by Miner's rule predictions $N_{\text{PMR}} = 10^4$.

The phenomenological Palmgren-Miner Rule (PMR) is studied under the framework of a SDV. The class of models which follow the PMR is identified and it is proven that the PMR constitutes an upper bound for random fatigue life predictions as demonstrated in Figure 1, i.e.,

$$N_{\text{PMR}}\{\sigma\} \geq N\{\sigma\}. \tag{5}$$

Multi-axial two-level tests [5] do not exhibit the symmetry of SDV models, thus this phenomenon requires a higher number of damage variables. We generalized the characterization of damage accumulation from two-level fatigue to three-level and captured a richer two damage variable evolution. The three-level tests are demonstrated on a simple theoretical model [6].

CONCLUSIONS

An analytical study was performed on the family of SDV fatigue models. A new integral formula for life prediction under random loads was developed and implemented on two well-known models [1, 4]. Common characteristics were identified and studied, and may serve as general guidelines for future modeling. Five main characteristics are:

1. Two-level fatigue life can be predicted solely by the one-level damage evolution.
2. H-L and L-H envelopes are anti-symmetrical.
3. Two-level envelopes are confined to the range $0 \leq n_1 \leq N_1, 0 \leq n_2 \leq N_2$.
4. Fatigue life under any multiple loading protocol can be found on the basis of two-level fatigue envelopes.
5. The PMR prediction constitutes an upper bound for SDV fatigue life predictions under random loads.

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