What Happens Beyond Drucker's Proposition When Damage Occurs

- continuous bifurcation, damage localization and catastrophic rupture

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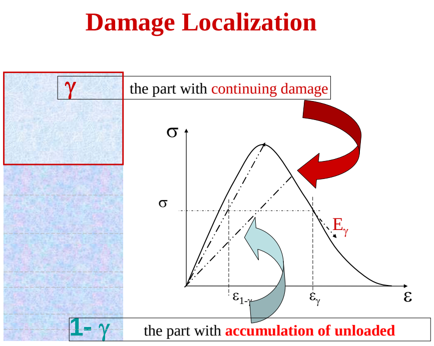
Summary The following manuscript introduces the appearance of damage localization in a sample beyond the strain at peak load, in accord with the Elastic Statistically-Brittle (ESB) model. Though in terms of continuum damage mechanics, catastrophic rupture could not be forecasted, from the experimental and analytic studies we found that at least there are two possibilities to foresee catastrophic rupture.

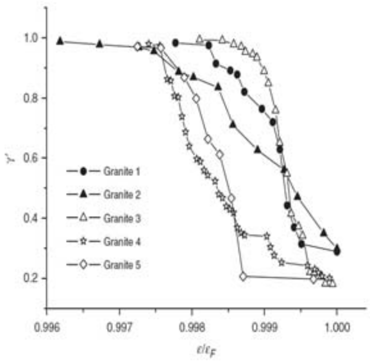
Everyone working in solid mechanics knows Drucker's proposition, beyond which material would become unstable. But, what may actually happen beyond Drucker's proposition in damaged materials? In accord with the model of Elastic Statistically-Brittle media (ESB), continuous bifurcation can appear beyond the strain at peak load. With increasing deformation a damaged sample could be split into two parts in series: a continuing damage zone and a zone with elastic unloading. The damage zone can be continuously split into such two parts. Thus, damage would become localized and the nominal strain of the sample would depend on the accumulative outcome of damage localization γ as shown in the integral,

$$\varepsilon_{nominal}(\bar{\sigma}) = \frac{\bar{\sigma}}{\bar{E}_u(\bar{\sigma}_M)} - \bar{\sigma}\int_{\bar{\sigma}_M}^{\bar{\sigma}} \gamma(\bar{\sigma}_u)\frac{\bar{E}_u{}'(\bar{\sigma}_u)}{\bar{E}_u{}^2(\bar{\sigma}_u)}\cdot d\bar{\sigma}_u \quad ,$$

where E_u denotes elastic unloading modulus and subscript u denotes the starting point of unloading.



(a)



(b)

Fig.1 Damage localization. (a) A sketch of damage localization and its relation to continuous bifurcation. (b) Experimentally measured damage localization of granite under compression.

Furthermore, a catastrophic rupture would be a self-sustainable process in energy aspect, for example, under stiff loading, the energy dissipated in the process of catastrophe can be fully supplied by the energy released from the sample itself, i.e., from the elastically unloading zone, regardless of external work.

$$\frac{\bar{\sigma}\bar{E}_u'(\bar{\sigma})}{\bar{E}_u^2(\bar{\sigma})}\gamma(\bar{\sigma}) - \int_{\bar{\sigma}}^{\bar{\sigma}_M} \frac{\bar{E}_u'(\bar{\sigma}_u)}{\bar{E}_u^2(\bar{\sigma}_u)}\gamma(\bar{\sigma}_u)d\bar{\sigma}_u \ \begin{matrix} > \\ \le \end{matrix}\ \frac{1}{\bar{E}_u(\bar{\sigma}_M)}, \qquad \begin{matrix} Stable \\ CR \end{matrix}$$

As the size of the damage localization zone becomes smaller, catastrophic rupture becomes more likely to appear.

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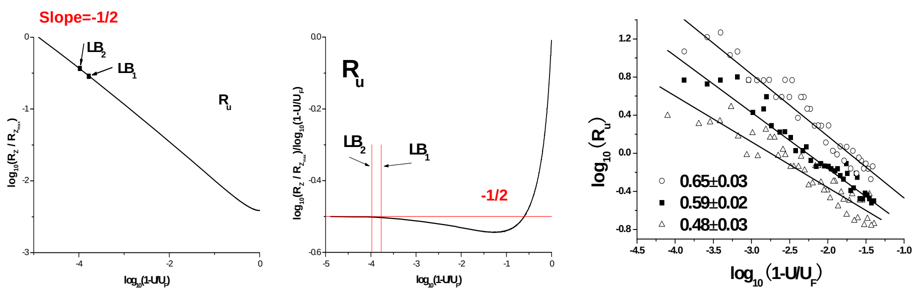
Unfortunately, the analysis of continuum damage mechanics shows that any scale of damage localization can satisfy the conservation laws. This implies that a catastrophic rupture could appear at any moment beyond peak load with a specific size of damage localization zone. The narrower the damage zone is, the nearer to the peak point the catastrophic rupture appears. In this sense, catastrophic rupture could not be forecasted in terms of continuum damage mechanics.

As a matter of fact, the uncertainty of such a catastrophic rupture is rooted in the complexity of the combination of the distributions of mesoscopic structures, stress-redistribution and damage evolution, etc. Hence, to predict the critical state of catastrophic rupture is far beyond the knowledge of available continuum damage mechanics.

However, in practice, from the experimental and analytic studies we found that at least there are two possibilities to foresee catastrophic rupture. The first is to monitor the damage localization and the second is to examine if a certain power law singularity (-1/2) appears or not.



(a) (b) (c)

Fig.2 -1/2 power law singularity emerging ahead of catastrophic rupture.

(a) -1/2 power law singularity emerges in a ESB model
(b) The lower bound of -1/2 power law singularity in the ESB model
(c) Experimentally measured power law singularity in granite under compression.

The applications of these ideas to some practical cases, like M9 and M8 earthquakes in Japan and Wenchuan, are quite encouraging.

Key Words: continuous bifurcation, damage localization, catastrophic rupture

References

[1] Bai Y. L., Wang H. Y., Xia M. F. and Ke F. J.: Statistical Mesomechanics of Solid, liking coupled multiple space and time scales, *Applied Mechanics Review* **58**: 372-388, 2005.

[2] Hao S. W., Xia M. F., Ke F. J. and Bai Y. L.: Evolution of Localized Damage Zone in Heterogeneous Media，*Int J Damage Mechanics* **19**: 787-804, 2010.