# MICRO INHOMOGENEOUS DEFORMATION AND FATIGUE LIFE OF A METAL UNDER SYMMETRIC STRAIN CYCLE

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*Summary* Considering the nonlinear kinematic hardening mechanism of crystal plasticity slip and applying a Voronoi polycrystal aggregate as a representative volume element (RVE) to simulate the microstructure of polycrystalline copper, hysteresis process was simulated with a crystal plasticity model under tension-compression symmetric cycle. Then the inhomogeneous evolution of the RVE's internal micro strain was analyzed statistically at grain scale during cyclic loading. According to the results, it was found that the strain statistical standard deviation of the RVE increased with cycles. Moreover, if the criterion for fatigue failure was taken as that the standard deviation reached a limit value, the predicted fatigue life of the RVE would be in reasonable agreement with Manson-Coffin equation.

## APPROACH AND MODEL

Considering anisotropy, grain size, shape and random orientation in each grain of a polycrystalline copper, we used a polycrystal aggregation including 100 grains to analyze the plasticity of a copper. Due to there was enough grains, the apparent stress-strain response obtained by simulation for the RVE could be able to reveal macroscopic plasticity behaviour of a polycrystal; additionally, inhomogeneous mechanical properties and statistical properties induced by metal microscopic anisotropy and random structure could be investigated by incrementally calculation.
The crystal plasticity model originally established by Hill and Rice [1] was applied, which was improved by introducing the slipping back stresses and in the modified model the nonlinear kinematic hardening was taken into account for its evolution when slip-system slipping [2], to the calculation of internal stress and strain in the crystal grain. The aggregation filled with Voronoi polyhedron was applied to simulate the polycrystalline copper, which could reflect the random grain size, shape and the orientation of crystal-axis. The stress-strain responses, including hysteresis loops and cyclic hardening, of polycrystalline copper under tension-compression symmetric cycles with different amplitudes were simulated, in order to research the evolutions of inhomogeneity of mechanical properties and statistical characteristics with cycle increasing and the influence of the evolution to fatigue life. Through the statistical analysis in grain level to the internal inhomogeneous stress and strain of the copper under cyclic loading, the relation between fatigue failure and the statistical parameters for stress and strain of the copper was discussed.

### Crystal visco-plasticity model

Considering the Bauschinger effect in a single crystal under reciprocating loading, a back stress item was introduced to Hutchinson's power law. Then, we have [2]

$$\dot{\gamma}^{(\alpha)} = \dot{\gamma}_0 \,\mathrm{sgn}(\tau^{(\alpha)} - x^{(\alpha)})\left|\frac{\tau^{(\alpha)} - x^{(\alpha)}}{g^{(\alpha)}}\right|^k, \quad \dot{g}^{(\alpha)}(\gamma) = \sum_{\beta}^{n} h_{\alpha\beta}(\gamma)\left|\dot{\gamma}^{(\beta)}\right|, \quad \gamma = \int \sum_{\beta}^{n}\left|d\gamma^{(\beta)}\right| \tag{1}$$

where $\tau^{(\alpha)}$ denotes the resolved shear stress of $\alpha$ slip system in a material point, $x^{(\alpha)}$ the back stress of $\alpha$ slip system, $\dot{\gamma}^{(\alpha)}$ the resolved shear strain rate of $\alpha$ slip system, $\dot{\gamma}_0$ the reference shear strain rate (taken to be a material constant), $g^{(\alpha)}$ slip hardening function of $\alpha$ slip system; $h_{\alpha\beta}(\gamma)$ denotes the slip hardening modulus, $n$ the total number of all activated slip systems of a material point. The evolution of $x^{(\alpha)}$ is defined by the following equation [2]

$$\dot{x}^{(\alpha)} = a\dot{\gamma}^{(\alpha)} - c[1 - e_1(1 - \exp(-e_2\gamma))]x^{(\alpha)}\left|\dot{\gamma}^{(\alpha)}\right| - dx^{(\alpha)} \tag{2}$$

where $a$, $c$, $e_1$, $e_2$ and $d$ are taken to be the constants fitted by tests.

### Calculation for RVE of the polycrystal aggregation

In order to reflect structural characteristics and deformation feature of a polycrystal, the Voronoi polycrystal aggregation was applied to structure a RVE. This RVE was arranged randomly in size, shape and orientation with 100 single crystals. The plastic properties of the crystals were described by anisotropy crystal visco-plasticity model mentioned above. And this RVE was a cube filled with 27000 eight-node hexahedral elements，total 29791 nodes (see Fig.1). Its mechanical behavior was approximately equal in three coordinate directions as shown in Fig.1. The setting of boundary

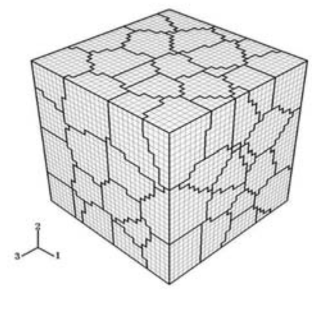

**Fig.1 Voronoi RVE**

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condition for the RVE made the external surface keep as plane in deforming, so the surface deformed seemed as homogeneous. The single crystal of copper has octahedron slip-system, elastic constants are: $C_{11}$ =136.4GPa, $C_{12}$ =98.334GPa, $C_{44}$ =61.074GPa; visco-plasticity parameter are, respectively, $\tau_0$ =13.9MPa, $\tau_s$ =30MPa, $h_0$ =96MPa, $a$ =20.6GPa, $c$ =1.42GPa, $e_1$ =0.41, $e_2$ =5.0, $d$ =0, and $\dot{\gamma}_0 = 1\times10^{-3}\mathrm{s}^{-1}$, $k$ =200.

## SIMULATION AND ANALYSIS

Loading was exerted to the RVE by strain controlling with tension-compression symmetric cycle, and a series of strain amplitude was applied and the half strain range, $\frac{1}{2}\Delta\varepsilon_t$, are respectively 0.3%，0.4%，0.5% and 0.6%. Then, the hysteresis loops with cycle increasing was obtained by calculation (see Fig.2). The results show that cyclic hardening and hysteresis loops would get stable in several ten cycles.

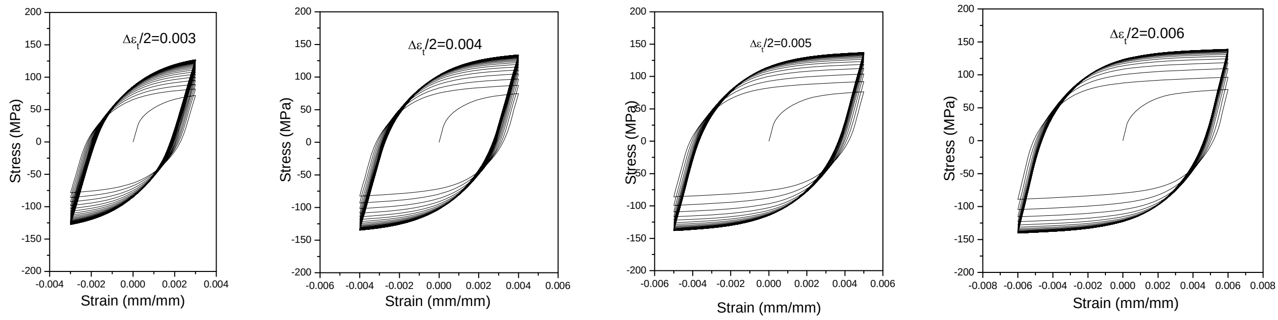


**Fig.2 Hysteresis loops by simulation**

### Statistical analysis of inhomogeneous deformation under symmetric strain cyclic

The statistical analysis of the stress and strain for the RVE was performed and the mathematical expectation and standard deviation were calculated in order to investigate the average and its deviation of relevant physical quantities in the whole RVE. The higher the standard deviation, the more inhomogeneous the physical quantities in material will be. The microscopic plastic strain and its distribution in RVE could be obtained by calculation, uneven deformation and its change in the polycrystal under cycle could be analyzed by statistical calculation. Then the statistical value for loading reached peak was obtained. It was found that the strains accorded with Gaussian approximately. In stable cycle, the peak strain remained the same, but the standard deviation for longitudinal strain would grow with increasing cycle.

### Low cyclic fatigue life was predicted with statistic standard deviation of strain

Fig.3 shows that statistic standard deviation of longitudinal strain, $\varepsilon_{33}$ , will grow with increasing cycles, it means that the microscopic strain in some area of the material will grow gradually during cycle. Thus, it is reasonable to assume that fatigue failure will happen when statistic standard deviation of strain reaches a certain value. If take the value equals to 0.0151, then the intersection of red line with the deviation curves in Fig.3 will give the prediction of fatigue life. Fig.4 shows the relation between fatigue life prediction and half strain amplitude, and it can be found that this relation is in reasonably agreement with Manson-Coffin's law.

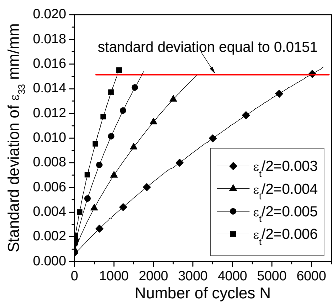


**Fig.3 Growing standard deviation of longitudinal strain**

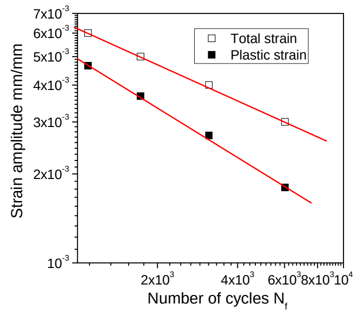


**Fig.4 Fatigue life prediction vs. half strain range**

## CONCLUSIONS

(1) Using presented method, the inhomogeneous micro-strain of a RVE during cycle can be analyzed in statistics.
(2) The statistical standard deviation of longitudinal strain of the RVE will grow with increasing cycle at tension peak, if it is judged the fatigue failure will happen when the statistical standard deviation reaches a certain value, then the predicted fatigue life will be in reasonably agreement with Manson-Coffin's law.

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