AN EXTENSION OF GURSON'S MODEL TO ARBITRARY ELLIPSOIDAL VOIDS

Komlanvi Madou[*a)], Jean-Baptiste Leblond[**]
[*] *Institut Jean Le Rond d'Alembert, Université Pierre et Marie Curie (Paris 6), Tour 55-65, 4 Place Jussieu, 75252 Paris Cedex 05, France*
[**] *Same address*

Summary Ductile fracture of metals occurs by nucleation, growth and coalescence of microvoids. Void growth is due to the plastic deformation of the surrounding matrix. The most classical model for plastic porous materials is the one introduced by Gurson [1] for spherical and cylindrical voids. It was later modified by Tvergaard and Needleman [2] , and extended to spheroidal (both prolate and oblate) voids by Gologanu et al. [3, 4, 5]. Nevertheless voids encountered in real materials are not always spheroidal. Our aim here is to further extend Gologanu et al.'s model to general ellipsoidal voids. A limit analysis of some ellipsoidal domain containing a confocal ellipsoidal void is performed. A Gurson-like approximate criterion is obtained and validated by finite element simulations of elementary cells.

INTRODUCTION

Gurson's classical model of the overall behavior of porous plastic solids [1] was derived from limit-analysis of a hollow sphere loaded through conditions of homogeneous boundary strain rate. Gologanu et al. [3, 4, 5] defined a more general model applicable to spheroidal, prolate or oblate voids. This was done by extending Gurson's limit-analysis to spheroidal domains containing confocal spheroidal voids. More general (non-spheroidal) ellipsoidal voids have not been considered within the approach initiated by Gologanu et al. [3, 4, 5], although such voids are common in practice. In this work, we extend Gologanu et al.'s approach to general ellipsoidal voids. The first step consists of using Leblond and Gologanu [6]'s velocity fields in a limit-analysis of some ellipsoidal domain containing a confocal ellipsoidal void, and loaded through conditions of homogeneous boundary strain rate. The second step is devoted to the determination of the yield criterion parameters. In the third step the yield function proposed is validated versus a number of numerical computations of yield surfaces of hollow cells of various ellipsoidal shapes.

LIMIT-ANALYSIS OF SOME ELLIPSOIDAL CELL CONTAINING A CONFOCAL ELLIPSOIDAL VOID

We consider an ellipsoidal cell made of some ideal plastic von Mises material with yield stress σ_0 in simple tension, and containing a confocal ellipsoidal void. The semi-axes of the inner ellipsoid (the boundary of the void) and the outer one (the boundary of the cell) are denoted a, b, c $(a > b > c)$ and A, B, C $(A > B > C)$ respectively. The volumes of these ellipsoids are $\frac{4\pi}{3}\omega$ $(\omega = abc)$ and $\frac{4\pi}{3}\Omega$ $(\Omega = ABC)$ and the porosity is $f = \frac{\omega}{\Omega}$. We use the ellipsoidal coordinates λ, μ, ν associated to a, b, c. The value of λ is 0 on the inner ellipsoid and Λ on the outer ellipsoid. Now load this cell through conditions of homogeneous boundary strain rate, $\mathbf{v}(\mathbf{r}) = \mathbf{D}.\mathbf{r}, \quad \mathbf{r} \in \partial\mathbf{\Omega}$. One may use the trial velocity field discovered by Leblond and Gologanu [6], satisfying conditions of homogeneous strain rate on confocals ellipsoids; there is a unique incompressible velocity field of this type compatible with the given value of $\mathbf{D}$. The limit-analysis based on such a field provides an upper estimate $\Pi^+(\mathbf{D})$ of the overall plastic dissipation defined by

$$\Pi^+(\mathbf{D}) = \quad \frac{\sigma_0}{\Omega}\int_{\lambda=0}^{\lambda=\Lambda} \langle d_{eq}(\mathbf{r})\rangle_{\mathcal{E}_\lambda} dv(\lambda), \qquad d_{eq}(\mathbf{r}) = \left(\tfrac{2}{3}\mathbf{d}(\mathbf{r}) : \mathbf{d}(\mathbf{r})\right)^{1/2} \tag{1}$$

where $\mathbf{d}(\mathbf{r})$ is the local strain rate tensor. In the expression $(1)_a$, $\langle d_{eq}(\mathbf{r})\rangle_{\mathcal{E}_\lambda}$ represents a suitable weighted average value of $d_{eq}(\mathbf{r})$ over the ellipsoidal surface of constant λ denoted here $\mathcal{E}_\lambda$.
An external estimate of the overall yield surface of the cell may be obtained from the equation

$$\mathbf{\Sigma} = \frac{\partial \Pi^+}{\partial \mathbf{D}}(\mathbf{D}) \tag{2}$$

Using several approximations, it becomes possible to evaluate the components of the macroscopic stress tensor $\mathbf{\Sigma}$ through differentiation, and finally eliminate the components of $\mathbf{D}$ in equation (2). One obtains a Gurson-like approximate overall criterion of the ellipsoidal cell of the form:

$$\frac{Q(\mathbf{\Sigma})}{{\sigma_0}^2} + 2(1+g)(f+g)\cosh\left(k\frac{\Sigma_h}{\sigma_0}\right) - (1+g)^2 - (f+g)^2 = 0 \tag{3}$$

where $Q(\mathbf{\Sigma})$ is a quadratic form of $\mathbf{\Sigma}$, Σ_h is a linear combination of its diagonal components,

$$g = \frac{\sqrt{a^2-c^2}(b^2-c^2)}{\Omega}, \tag{4}$$

and k is related to the overall macroscopic plastic dissipation for a hydrostatic loading.

a) Corresponding author. E-mail: madou@lmm.jussieu.fr

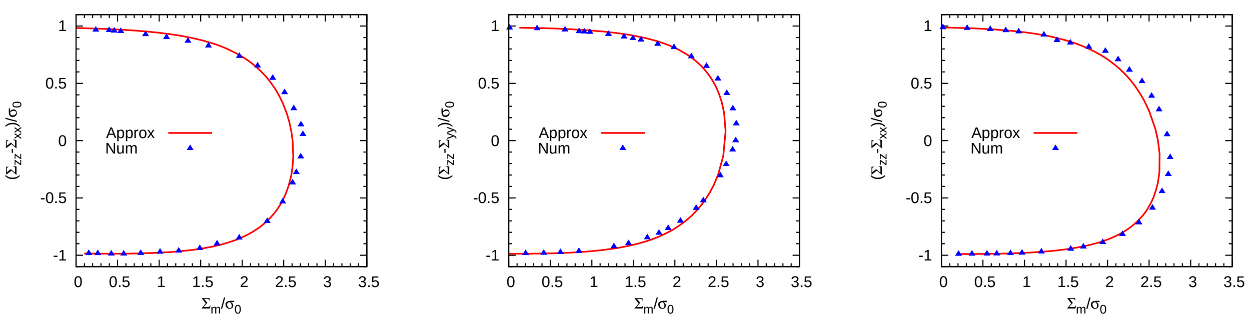
IMPROVEMENT OF OVERALL YIELD CRITERION

We use some "hybrid" approach to obtain the best possible expressions of the yield criterion parameters:

- For hydrostatic loadings, the limit-analysis is refined by performing micromechanical finite element computations in a number of significant cases, so as to replace Leblond and Gologanu [6]'s trial velocity field representing the expansion of the void by the exact, numerically determined one. This leads us to propose suitable approximate expressions for k, H_x, H_y, H_z.
- For deviatoric loadings, limit-analysis is dropped and direct use is made of some general rigorous results for non-linear composites derived by Willis [7]. One enforces coincidence of our yield function and the lower bound of Willis up to second order in Σ_h/σ_0. This determines the coefficients of the quadratic form $Q(\mathbf{\Sigma})$.

NUMERICAL VALIDATION OF THE YIELD CRITERION

We consider an ellipsoidal cell containing a confocal ellipsoidal void, with semi-axes in the proportions $10 : 2 : 1$ and a porosity of 0.01, loaded through conditions of homogeneous boundary strain. The full determination of the yield criterion would be a very heavy task, so we only study the traces of this surface in three orthogonal planes: $\{\Sigma_{xx} = \Sigma_{yy} \neq \Sigma_{zz}\}$, $\{\Sigma_{xx} \neq \Sigma_{yy} = \Sigma_{zz}\}$ and $\{\Sigma_{yy} \neq \Sigma_{xx} = \Sigma_{zz}\}$. (Figure 1)$_{a,b,c}$ shows that the numerical yield locus exhibits a slight corner on the "hydrostatic axis" unlike the approximate one. The presence of this corner is not surprising since the ellipsoidal void considered does not differ much from a circular cylindrical one, for which such a corner has long been known to exist (Pastor and Ponte-Castaneda [8]).



(a) $\Sigma_{xx} = \Sigma_{yy} \neq \Sigma_{zz}, \Sigma_{xy} = \Sigma_{xz} = \Sigma_{yz} = 0$ **(b)** $\Sigma_{xx} = \Sigma_{zz} \neq \Sigma_{yy}, \Sigma_{xy} = \Sigma_{xz} = \Sigma_{yz} = 0$ **(c)** $\Sigma_{yy} = \Sigma_{zz} \neq \Sigma_{xx}, \Sigma_{xy} = \Sigma_{xz} = \Sigma_{yz} = 0$

Figure 1. Traces in three orthogonal planes of the yield locus for a void with semi-axes in the proportions $10 : 2 : 1$ and a porosity of 0.01 - Numerical results (Num) and approximation proposed (Approx)

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