# ADAPTIVE FINITE ELEMENT MATERIAL POINT METHOD AND ITS APPLICATIONS IN IMPACT SIMULATION

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*Summary* As a meshless method, material point method (MPM) take advantages of both Lagrangian and Eulerian description for problems with extreme deformation. However, MPM is less accurate and less efficient than finite element method (FEM) for small deformation problems. Therefore, an adaptive finite element material point (AFEMP) method is proposed for impact simulation to take advantages of both methods.

## INTRODUCTION

The problems involving extreme deformation such as explosion, penetration and landslide problems are big challenges for mesh-based methods, so meshless methods have been proposed, such as the smoothed particle hydrodynamics method (SPH), the element free Galerkin method (EFG), just to name a few. However, these methods also suffer from their inherent shortcomings, and their efficiency is lower than that of finite element method (FEM). Hence, much effort has been devoted to couple meshless methods with FEM to take advantages of both methods. Rabaczuk reviewed such work in detail in [1].
Among meshless methods, material point method (MPM) [2] is very similar to FEM. In MPM, the material domain is discretized by a set of Lagrangian particles and the momentum equations are solved on an Eulerian background grid. In each time step, the particles are attached with the background grid so the grid serves as a finite element discretization to the material domain. Unlike other meshless method, the trial functions of MPM are same to that of FEM. Therefore, MPM shows some advantages over other meshless methods in efficiency and stability [3]. However, the efficiency of MPM is lower than FEM due to the mappings between background grid and particles, and the accuracy of particles quadrature used in MPM is less than that of Gauss quadrature used in FEM. Hence, Zhang [4] developed an explicit material point finite element method (MPFEM) to take advantages of both FEM and MPM, but the conversion of FEM to MPM is not flexible in MPFEM. In this paper, an adaptive finite element material point (AFEMP) method is proposed, which discretizes the material region by finite elements, and the distorted elements are adaptively converted into MPM particles based on criteria.

## AFEMP METHOD

### Discretization

In AFEMP method, a material region $\Omega$ is firstly discretized by finite elements as shown in Figure 1 (b). During the simulation process, the distorted element is converted into MPM particles as shown in Figure 1 (c) where regular background grid is used to solve the momentum equations.

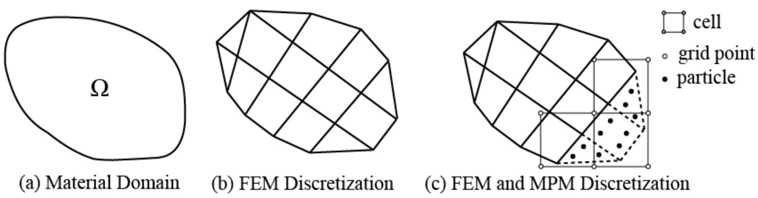


Figure 1: a material domain with different discretization

### Conversion of element to particles

Conversion algorithm of element to particles is proposed to avoid finite element distortion. Plastic strain or critical time step size during the solution process can be used as criteria with a user-specified threshold value. As shown in Figure 2, there are two elements A and B which are designated as candidates for conversion to particles. For conservation of mass, momentum, and energy, one element is converted into four MPM particles whose nature coordinates are taken as ($\pm 0.5$, $\pm 0.5$) and whose velocities are set as those of the adjacent FE nodes. After conversion, the elements A and B and FE node c are deleted, while four particles for each element are added. In 3D problems, one hexahedral element is replaced by 8 particles in a similar way.

### Connection between FEM and MPM

The connection between FEM part and MPM part is handled by transition nodes based on background grid in MPM framework as shown in Figure 3. For transition nodes, their momentum equations are solved on the background grid by

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mapping their masses and momentums and nodal forces to background grid. Before calculate the elements strain, the velocity of the transition nodes must be reset by mapping the velocity from the background grid back to it to guarantee the consistency of the velocity field used for the incremental strain calculation of element and particles. And the position and velocity of transition nodes are updated by the result of momentum equations of background grid nodes.

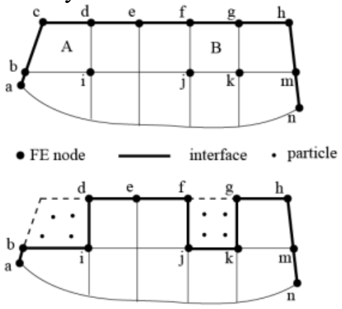



Figure 2: Conversion scheme

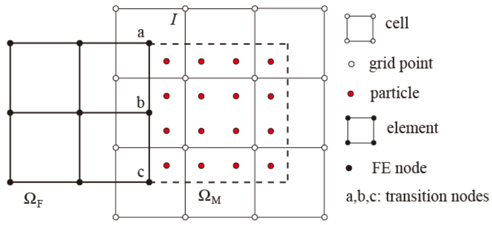
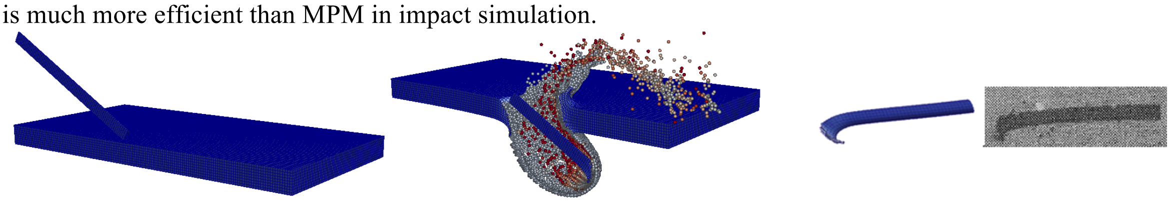



Figure 3: Illustration for the connection between FEM and MPM

## NUMERICAL EXAMPLE

The WHA long rod projectile penetration experiment conducted by Holmberg [5] is studied by this method. In the experiment the projectile hits steel armour plate under an angle of 60 degree with initial velocity of 2500m/s. The length and the diameter of projectile are 75mm and 5mm, respectively, while the size of the target is 200×100×9mm. Due to symmetry, half model is used as shown in Figure 4 (a), where hexahedral elements are used for the whole material domain. Distorted elements are gradually converted into MPM particles as shown in Figure 4 (b). The projectile's residual length ratios obtained by AFEMP and experiment are 0.72 and 0.76, respectively, while the projectile's residual velocity ratios obtained by AFEMP and experiment are 0.97 and 0.99, respectively. The residual part of the projectile obtained by AFEMP is also in good agreement with the experiment result as shown in Figure 4 (c). Furthermore, the computational cost required by AFEMP and MPM are listed in the Table 1, which shows that AFEMP is much more efficient than MPM in impact simulation.



(a) (b) (c)

Figure 4: (a) Discretization model; (b) Conversion during the simulation process; (c) Comparison of residual part of projectile between simulation result and experiment data

Table1: Computational cost required by MPM and AFEMP

| | Maximum time step/ $\mu$s | Minimum time step/ $\mu$s | Total steps | Cost/ s |
|---|---|---|---|---|
| MPM | 6.45e-2 | 3.71e-2 | 1180 | 2241 |
| AFEMP | 3.67e-2 | 1.52e-2 | 2208 | 838 |

## CONCLUSION

In this paper, an adaptive finite element material point method is developed for extreme deformation problems. Bodies are first discretized by finite elements, and then the distorted elements are adaptively converted into MPM particles based on a given criteria. The interaction between the FEM part and MPM part is handled based on the background grid in MPM framework. Therefore, AFEMP methods take the advantages of both FEM and MPM, and its efficiency is higher than both MPM and FEM as shown in the numerical example.

## ACKNOWLEDGMENT

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