NUMERICAL SIMULATION ON PAINLEVÉ PARADOX DURING FRICTIONAL IMPACT OF A FLEXIBLE BAR

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Summary Painlevé paradox is a well-known problem in non-smooth dynamics. In the present paper, a completely flexible body model incorporated with finite element method (FEM) is proposed to analyze the problem. Numerical results show that considering the compliance of the rest of the bar will decrease the critical value of μ, in other words, the bar has a larger compliance will result in self-locking phenomenon with a less μ. Besides, the peak value of normal contact force F_3 and normal contact impulse P_3 are constant as system parameters in the region of self-locking.

INTRODUCTION

For a large coefficient of friction and sliding contact between rigid bodies in a configuration that is asymmetric with respect to the common normal for the point of contact, equations of motion from rigid-body dynamics that incorporate the Amontons-Coulomb law of friction can result in a range of configurations where a solution either does not exist or is nonunique-this has been termed Painlevé paradox [1]. Recently, it has been formulated as a Linear Complementarity Problem (LCP) in order to clearly identify the range of parameters where the Painlevé paradox occurs [2-4]. Till now, two main methods were developed To resolve this paradox. 1) “Shock assumption”. It assumes the sliding is a instantaneous shock process. But contact forces and stick-slip motion can not be captured. 2) “Hybrid model” [5-6]. The compliance of the local contact zone is considered, but the rest part is rigid. It is useful for hard body, but its correctness for a non-hard body deserves to be doubted. The present paper goes beyond the formers researches in that it considers both of the compliance of local contact zone and the rest of the structure. A completely flexible body model incorporated with finite element method (FEM) is proposed to analyze the classical problem of Painlevé paradox. The numerical results compare with those of completely rigid body model and hybrid model.

MECHANICAL MODEL

There is a rigid homogeneous slender bar with one end C sliding on a rigid half-space (see Figure 1). The length of the bar is $2l$, the mass is m , the moment of inertia about the center of mass is $J_G = ml^2/3$ and the angle between the bar and the normal line of the tangential plane is θ . When the center of mass of the bar is located at the left of the normal line through the contact point C as shown in Figure 1, θ is negative, whereas it is positive if the center of mass is to the right of the normal. The Amontons-Coulomb friction law is assumed to hold at the contact point C with a constant friction coefficient $\mu > 0$.

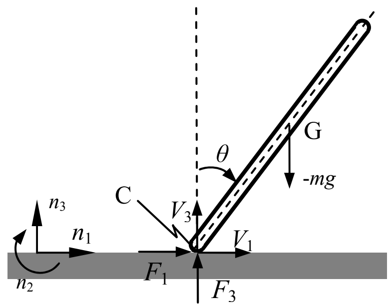

Figure 1 Bar with one end sliding on a rigid half-space

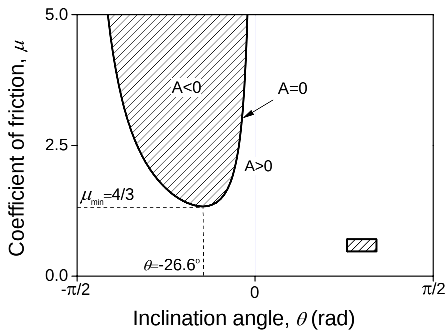

Figure 2 Region of Painlevé paradox when sliding is positive

Completely rigid body model and hybrid model

Based on completely rigid body model and Amontons-Coulomb friction law, to solve this unilateral constraint problem for a period of sliding in a positive direction, it has been reduced to a linear complementarity problem (LCP) [2-3],

$$\ddot{y}_C = AF_3 + b \geq 0 \,, \quad F_3 \geq 0 \,, \quad F_3 \ddot{y}_C = 0 \tag{1a}$$

with

$$A(\theta,\mu) = [1 + 3\sin\theta(\sin\theta + \mu\cos\theta)]/m \,, \quad b(\theta,\dot{\theta}) = l\dot{\theta}^2\cos\theta - g \tag{1b}$$

where $\ddot{y}_C$ and F_3 are normal relative acceleration and normal contact force at contact point. Inconsistency (LCP has no solution) or indeterminacy (non-uniqueness) of the solution is termed Painlevé paradox [2]. From the above formula, the Painlevé paradox region ($A \leq 0$) can be obtained (see Figure 2). As the incline angle is –π/4, the critical value of coefficient of friction μ is 5/3, i.e., if the coefficient of friction is larger than 5/3, Painlevé paradox occurs. Some

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researchers [6-7] proposed a hybrid model to resolve this conundrum. The self-locking phenomenon was captured, and they [7] found that the Self-locking region is just the same as the Painlevé paradox region.

Completely flexible body model

In present paper, a completely flexible body model is proposed. The bar is not modelled as a rigid body, but an elastic bar with one spherical contact end. The commercial FEM codes LS-DYNA is used to simulated the dynamic behaviour as the bar sliding on a rough rigid half-space. The three-dimensional elastic elements and rigid elements are used to discretize the bar and rigid half-space, respectively. A very dense discretization of the contact zone is necessary to capture the contact forces and contact stress precisely. The rest part of the bar can be discretized by large size elements. The contact algorithm is chosen as automatic surface to surface. The time scale factor is chosen as 0.5.

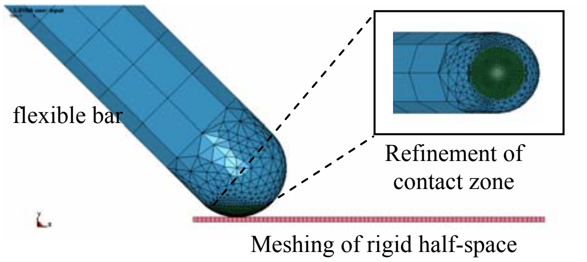

Figure 3 Finite element model for bar oblique impacts on rigid half-space

NUMERICAL RESULTS

The bar has length 1m, Young's Modulus 210GPa, density 7800kg/m^3, area of cross-section 0.0001m^2 and ration of Poisson is 0.27. The configuration angle $\theta=-\pi/4$. To focus on studying the dynamics behaviour to Painlevé paradox, the system parameters should locate in the region of Painlevé paradox. At time $t=0$, the bar is translating with uniform velocity $V_1(0)=-3.5\text{m/s}$ and it has no normal relative velocity at the contact point, i.e., $V_3(0)=0\text{m/s}$.

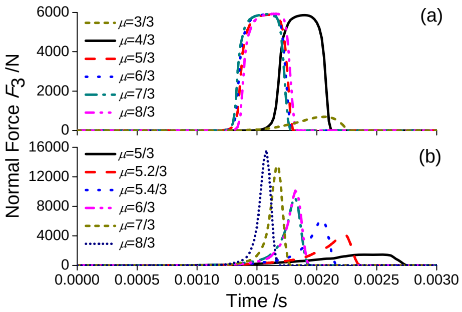

Figure 4 Normal contact forces: (a) solution of completely rigid body model; (b) solution of hybrid model

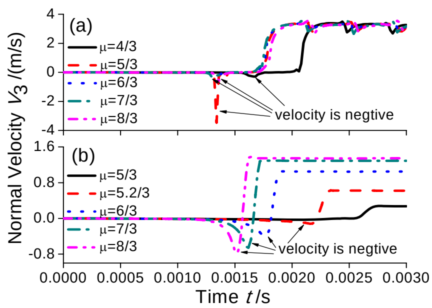

Figure 5 Normal relative velocity at contact point: (a) solution of completely rigid body model; (b) solution of hybrid model

Figure 4 are the normal contact forces F_3. The numerical results show that considering the compliance of the rest of the bar will decrease the critical value of μ. The peak value of F_3 and normal contact impulse P_3 is a constant as μ>4/3, Or else the peak value of F_3 will increase as increasing μ, but P_3 is also maintain a constant. Figure 5 is the normal relative velocity V_3 at the contact point. It can be found that for completely flexible body model , V_3 is negative as μ>4/3 (see Figure 5(a)); For hybrid model, V_3 is equal zero as μ=5/3. V_3 is negative as μ>5/3 (see Figure 5(b)). The numerical results show that the Self-locking region moves forward as consider the compliance. In the other words, the bar has a larger compliance will results in Self-locking phenomenon with a less μ.

CONCLUSIONS

A completely flexible body model incorporated with finite element method (FEM) is presented to analyze classical problem of Painlevé paradox. Results show that considering the compliance of the rest of the bar will decrease the critical value of μ, the self-locking region moves forward as consider the compliance. In the other words, the bar has a larger compliance will result in Self-locking phenomenon with a less μ. Besides, the peak value of F_3 and normal contact impulse P_3 maintain constant as the system parameters locating in the region of Self-locking.

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