A SECOND-ORDER CONE COMPLEMENTARITY APPROACH FOR NUMERICAL SOLUTION OF 3D ELASTOPLASTIC CONTACT PROBLEMS WITH ORTHOTROPIC FRICTION LAW

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Summary 3D elastoplastic contact problems with orthotropic friction law are treated by a second-order cone complementarity approach. Firstly, the 3D contact condition with orthotropic friction law and the elasto-plastic constitutive equation are both reformulated as second-order cone complementarity conditions. Secondly, the governing equations of the 3D elastoplastic contact problems with orthotropic friction law are formulated as a second-order cone complementarity problem (SOCCP) via the parametric variational principle and the finite element method. Finally, a semi-smooth Newton algorithm is suggested to solve the obtained SOCCPs. Numerical results show the validity of the model.

INTRODUCTION

Contact problems are of particular importance in various engineering applications such as structure-structure interaction, machine design and metal forming. They are hard to solve because of their geometric and material nonlinearities. It is a general phenomenon that the nonlinear properties in both material form and contact form will occur at the same time. One of the difficulties in solving the problem lies in the determination of the tangential slip states at the contact points. A great amount of computational efforts is needed so as to obtain high accuracy numerical results [1,2]. In this paper, we develop a new formulation and solution algorithm by using the approach of second order cone complementarity problems (SOCCPs) for 3D elastoplastic contact problems with orthotropic friction law. The SOCCP formulation avoids the polyhedral approximation to the elliptical Coulomb friction cone and the incremental linearization to the nonlinear yield function so that the problem to be solved has much smaller size and the solution has better accuracy.

SOCCP FORMULATION FOR 3D ELASTOPLASTIC CONTACT PROBLEMS WITH ORTHOTROPIC FRICTION LAW

Consider two candidate contact bodies $\Omega^{(1)}$ and $\Omega^{(2)}, \Omega = \Omega^{(1)} \bigcup \Omega^{(2)}$. Let the common elastoplastic contact surface be denoted by S_c. We denote the principal orthotropic friction axes on the tangential plane at a point on Sc locally by $\tau_1$and τ_2, and let $p_{\tau 1}$, $p_{\tau 2}$ and $\varepsilon_{\tau 1}$, $\varepsilon_{\tau 2}$ be the components of tangential contact traction $\boldsymbol{p}_\tau$ and tangential contact relative displacement $\boldsymbol{\varepsilon}_\tau$ along these axes. The corresponding coefficients of friction are denoted by μ_1 and μ_2. Let p_n denote normal contact force.

SOCCP formulation of the 3D contact condition with Coulomb's type orthotropic friction law

We develop a new SOCCP formulation of the static 3D contact condition with Coulomb's type orthotropic friction law in incremental form as follows

$$\left.\begin{aligned} &\mathrm{d}\boldsymbol{p}_\mathrm{c} = \boldsymbol{D}_\mathrm{c}\left(\mathrm{d}\boldsymbol{\varepsilon}_\mathrm{c} - \tilde{\boldsymbol{Q}}\boldsymbol{\lambda}_\mathrm{c}\right) \\ &\boldsymbol{v}_\mathrm{c} = \tilde{\boldsymbol{W}}\boldsymbol{D}_\mathrm{c}\left(\mathrm{d}\boldsymbol{\varepsilon}_\mathrm{c} - \tilde{\boldsymbol{Q}}\boldsymbol{\lambda}_\mathrm{c}\right) \\ &\boldsymbol{v}_\mathrm{c} \in \kappa^3 \times \kappa^1, \boldsymbol{\lambda}_\mathrm{c} \in \kappa^3 \times \kappa^1 \\ &\boldsymbol{v}_\mathrm{c}^\mathrm{T}\boldsymbol{\lambda}_\mathrm{c} = 0 \end{aligned}\right\} \tag{1}$$

where $\boldsymbol{p}_\mathrm{c} = \left(p_{\tau 1}, p_{\tau 2}, p_\mathrm{n}\right)^\mathrm{T}$, $\boldsymbol{\varepsilon}_\mathrm{c} = \left(\varepsilon_{\tau 1}, \varepsilon_{\tau 2}, \varepsilon_n\right)^\mathrm{T}$, $\boldsymbol{\lambda}_\mathrm{c} = \left(\lambda_{\tau 0}, \lambda_{\tau 1}, \lambda_{\tau 2}, \lambda_\mathrm{n}\right)^\mathrm{T}$ denotes the flow parameter vector for contact , κ^n is the second order cone defined as

$$\kappa^n := \left\{\left(x_1, \boldsymbol{x}_2\right) \in \Re \times \Re^{n-1} \mid \left\|\boldsymbol{x}_2\right\| \le x_1\right\}.$$

SOCCP formulation of the J_2 plasticity

We develop a new SOCCP formulation of the J_2 plasticity with combined linear isotropic and kinematic hardening laws in incremental form as follows.

$$\mathrm{d}\boldsymbol{\sigma} = \boldsymbol{D}\left(\mathrm{d}\boldsymbol{\varepsilon} - \boldsymbol{R}\boldsymbol{\lambda}_p\right) \tag{2a}$$

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$$\begin{cases} \boldsymbol{\lambda}_p \in \mathcal{K}^7 \\ \boldsymbol{v} := \begin{Bmatrix} \sqrt{\dfrac{2}{3}}\left(\sigma_s - q_1^0\right) \\ \boldsymbol{Q}\left(\boldsymbol{\sigma}^0 - \boldsymbol{q}_2^0\right) \end{Bmatrix} + \begin{Bmatrix} \boldsymbol{0}_{1\times6} \\ \boldsymbol{QD} \end{Bmatrix} \mathrm{d}\boldsymbol{\varepsilon} + \begin{bmatrix} \dfrac{2}{3}H_{iso} & \boldsymbol{0}_{1\times(nq-1)} \\ \boldsymbol{0}_{6\times1} & \boldsymbol{Q}\left(H_{kin}\boldsymbol{I}_{6\times6} + \boldsymbol{D}\right)\boldsymbol{Q}^{\mathrm{T}} \end{bmatrix} \boldsymbol{\lambda}_p \in \mathcal{K}^7 \\ \boldsymbol{\lambda}_p^{\mathrm{T}}\boldsymbol{v} = 0 \end{cases} \tag{2b}$$

where $\boldsymbol{R}=-\left[\boldsymbol{0}_{1\times6}\ \ \boldsymbol{Q}^{\mathrm{T}}\right]$, $\boldsymbol{D}$ is the Hooke's elastic matrix, $\boldsymbol{q} := \left(q_1, \boldsymbol{q}_2\right)$ are the hardening internal variables in stress space, H_{iso} is the isotropic hardening moduli, H_{kin} is the kinematic hardening moduli, $\boldsymbol{I}$ is an identical matrix. $\boldsymbol{\lambda}_p$ denotes the flow parameter vector for plasticity. The matrixes in (1) and (2) are as follows.

$$\boldsymbol{D}_{\mathrm{c}} = \begin{bmatrix} D_{\tau1} & 0 & 0 \\ 0 & D_{\tau2} & 0 \\ 0 & 0 & D_{\mathrm{n}} \end{bmatrix}, \quad \tilde{\boldsymbol{Q}} = \begin{bmatrix} 0 & \dfrac{1}{\mu_1} & 0 & 0 \\ 0 & 0 & \dfrac{1}{\mu_2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \tilde{\boldsymbol{W}} = \begin{bmatrix} 0 & 0 & -1 \\ \dfrac{1}{\mu_1} & 0 & 0 \\ 0 & \dfrac{1}{\mu_2} & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad \boldsymbol{Q} = \begin{bmatrix} \dfrac{1}{\sqrt{3}} & -\dfrac{1}{\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & \dfrac{1}{\sqrt{3}} & -\dfrac{1}{\sqrt{3}} & 0 & 0 & 0 \\ -\dfrac{1}{\sqrt{3}} & 0 & \dfrac{1}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{2} \end{bmatrix}$$

where $D_{\tau1}, D_{\tau2}$ and D_{n} are large numbers named regularization parameters.

Finite element model of 3D elastoplastic contact problems with orthotropic friction law

Under the assumption of small deformations, a parametric variational principle[1] of 3D elastoplastic contact problems with orthotropic friction law can be established. Furthermore, by using the finite element method, we finally derived the following linear SOCCP formulation for the numerical solution of incremental 3D elastoplastic contact analysis problems.

$$\begin{cases} \boldsymbol{K}\mathrm{d}\bar{\boldsymbol{u}} - \Phi\bar{\boldsymbol{\lambda}} = \mathrm{d}\boldsymbol{p} \\ \bar{\boldsymbol{v}} = \boldsymbol{C}\mathrm{d}\bar{\boldsymbol{u}} - \boldsymbol{U}\bar{\boldsymbol{\lambda}} + \boldsymbol{d} \\ \bar{\boldsymbol{\lambda}}^{\mathrm{T}}\bar{\boldsymbol{v}} = 0, \bar{\boldsymbol{\lambda}} \in \boldsymbol{\mathcal{K}}, \bar{\boldsymbol{v}} \in \boldsymbol{\mathcal{K}} \end{cases} \tag{3}$$

where K, Φ, U, C are the matrixes with constant numbers, only the vectors $\mathrm{d}\boldsymbol{p}$ and $\boldsymbol{d}$ need be updated at each incremental loading step. Equation (3) can be solved directly by the semi-smooth Newton algorithm developed in the mathematical field for the SOCCPs[3].

CONCLUSIONS

The elastoplastic contact problem with orthotropic friction law is studied in this paper. With the orthotropic friction law and the elastoplasticity constitutive law being rewritten as second order cone complementarity formulations, the problem is reduced to a standard SOCCP. The SOCCP has a simple mathematical expression, as well as effectively overcomes the disadvantages of the explicitly linearization approximation to Coulomb friction cone and the yield function when using the linear complementarity approach.

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