EXTENDED JKR THEORY ON ADHESIVE CONTACT BETWEEN LAYERED ELASTIC SPHERES

Seung Tae Choi[a)]

School of Mechanical Engineering, University of Ulsan, Ulsan 680-749, Republic of Korea

Summary The conventional JKR theory was extended to adhesive contact between layered elastic spheres. Elasticity problems of axisymmetric indentation without adhesion on a layered elastic half-space were revisited, with which the force-depth relations were obtained in a simple form. By using the solutions of the axisymmetric indentations, the contact between layered elastic spheres was analyzed, in which the adhesion energy between the spheres was taken into account. The obtained result differs from that of the conventional JKR theory only in several non-dimensional factors depending on elastic constants and geometric parameters. Also, the energy release rate at the debonding between layered elastic spheres was expressed in terms of pull-off force, elastic constants, and geometric parameters.

INTRODUCTION

Adhesive contact between coated materials, unlike bulk materials, entails difficulties in obtaining the adhesion properties of the coating materials due to the effect of substrates. For example, if the theory of adhesive contact between elastic bodies, known as the JKR theory developed by Johnson et al. [1], is used for spherical indentation onto an elastic film bonded to a substrate, only qualitative results are provided, unless the thickness of the film is much larger than the contact radius of the spherical tip. Several researchers [2-7] have studied adhesive contact between an axisymmetric tip and an elastic film on a rigid or elastic substrate. However, no JKR-type theory is available for adhesive contact between layered spheres, even if it is known that the JKR theory can be developed by appropriately superposing the solutions of a flat indentation and a spherical indentation [1, 8]. In this study, the conventional JKR theory is extended to the adhesive contact between layered spheres.

EXTENTION OF THE CONVENTIONAL JKR THEORY

The adhesive contact between layered elastic spheres, as shown in Fig. 1, analyzed by following the same procedure as described by Maugis [8]. The two spheres with radii R_L and R_U are assumed to be linear elastic and have elastic coatings with thicknesses h_L and h_U, respectively. Under a compressive force F, the area of contact becomes a circular region with radius a. The relative displacement δ between the centers of the two spheres under the applied force F is the sum of the displacements of the two spheres, that is, $\delta = \delta_L + \delta_U$. The details of the analysis are not shown here, but the results are as follows:

$$\overline{F} = \frac{K_f\{x\}}{K}\overline{a}\left[\left(\frac{K_s\{x\}}{K_f\{x\}}\chi_s\{x\} + \gamma\{x\}\right)\overline{a}^2 - \sqrt{\frac{2K\overline{a}}{K_f^*\{x\}} + \left(\gamma\{x\}\overline{a}^2\right)^2}\right], \tag{1}$$

$$\overline{\delta} = \left(3\chi_s\{x\} + 2\gamma\{x\}\right)\overline{a}^2 - 2\sqrt{\frac{2K\overline{a}}{K_f^*\{x\}} + \left(\gamma\{x\}\overline{a}^2\right)^2}\ . \tag{2}$$

Here $\overline{F} = F/3\pi\omega R$, $\overline{\delta} = \delta(3K^2/\pi^2\omega^2 R)^{1/3}$, and $\overline{a} = a(K/3\pi\omega R^2)^{1/3}$ are the normalized contact force, normalized relative displacement, and normalized contact area, respectively, and $1/K = (1-\nu_1^2)/E_1 + (1-\nu_1^2)/E_1$ and $1/R = 1/R_L + 1/R_U$. Also, in Eqs. (1) and (2), the parameters, $K_f\{x\}$, $K_f^*\{x\}$, $K_s\{x\}$, $\chi_s\{x\}$, and $\gamma\{x\}$, are non-dimensional parameters depending on elastic constants and normalized contact radius $x \equiv a/h_L$. The equations (1) and (2) differ from those of the conventional JKR theory [1], only in those non-dimensional factors. For example, the parameter $K_s\{x\}$ has the relation $1/K_s\{x\} \equiv (1-\nu_1^2)/[E_1\Phi_s(x;\alpha_{12},\beta_{12})] + (1-\nu_3^2)/[E_3\Phi_s(xh_L/h_U;\alpha_{34},\beta_{34})]$, in which $\Phi_s(a/h;\alpha_{12},\beta_{12})$ can be obtained as shown in Fig. 2.

RESULTS AND DISCUSSION

Equations (1) and (2) expressing the extended JKR theory developed in this study can be used to obtain the relations between contact force, relative displacement, and contact area. Figs. 3 and 4 show the plots of normalized contact area vs. normalized contact force and normalized contact force vs. normalized relative displacement, respectively.

a) Corresponding author. Email: stchoi@ulsan.ac.kr.

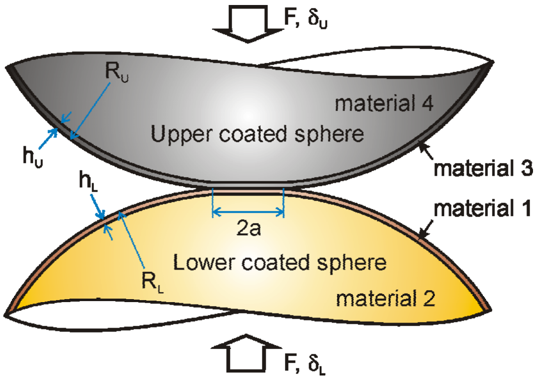

Fig. 1. Contact of layered elastic spheres with adhesion.

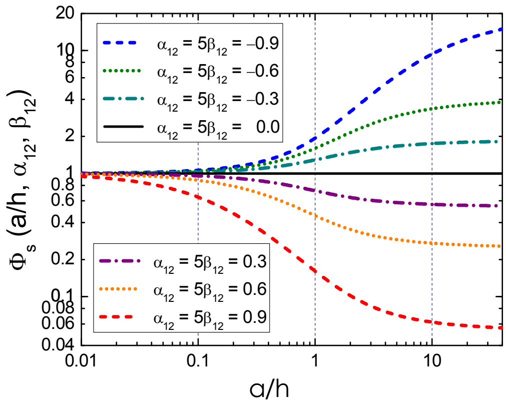

Fig. 2. The plots of the non-dimensional parameter $\Phi_s(a/h;\alpha_{12},\beta_{12})$.

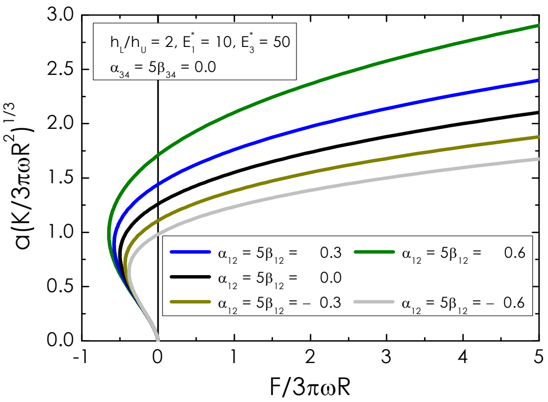

Fig. 3. The plot of normalized contact area versus normalized contact force.

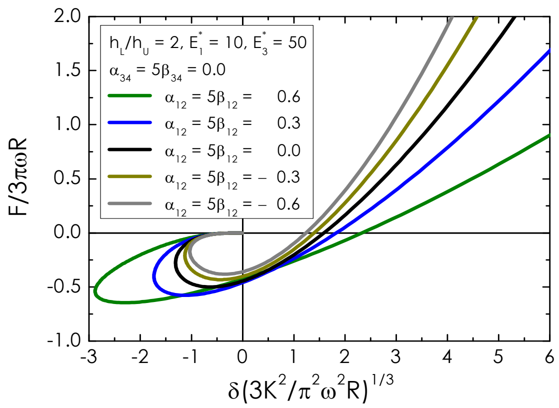

Fig. 4. The plot of normalized contact force versus normalized relative displacement.

CONCLUSIONS

The extended JKR theory was developed in this study for adhesive contact between layered elastic spheres. The obtained result differs from that of the conventional JKR theory only in several non-dimensional factors depending on elastic constants and geometric parameters. Also, the energy release rate at the debonding between layered elastic spheres was expressed in terms of pull-off force, elastic constants, and geometric parameters. The results of the present study are useful in exploring the adhesion between thin elastic films especially at micro and nano scales.

References

[1] Johnson K. L., Kendall K., Robert A. D.: Surface energy and the contact of elastic solids. *Proc. R. Soc. Lond. A* **324**: 301-313, 1971.
[2] Hayes W. C., Keer L. M., Herrmann G., Mockros L. F.: A mathematical analysis for indentation tests of articular cartilage. *J. Biomechanics* **5**: 541-551, 1972.
[3] Yu H. Y., Sanday S. C., Rath B. B.: The effect of substrate on the elastic properties of films determined by the indentation test-axisymmetric Boussinesq problem. *J. Mech. Phys. Solids* **38**: 745-764, 1990.
[4] Sridhar I., Johnson K. L., Fleck N. A.: Adhesion mechanics of the surface force apparatus. *J. Phys. D: Appl. Phys.* **30**: 1710-1719, 1997.
[5] Johnson K. L., Sridhar I.: Adhesion between a spherical indenter and an elastic solid with a compliant elastic coating. *J. Phys. D: Appl. Phys.* **34**: 683-689, 2001.
[6] Yang F.: Thickness effect on the indentation of an elastic layer. *Materials Science and Engineering A* **358**: 226-232, 2003.
[7] Sridhar I., Zheng Z. W., Johnson K. L.: A detailed analysis of adhesion mechanics between a compliant elastic coating and a spherical probe. *J. Phys. D: Appl. Phys.* **37**: 2886-2895, 2004.
[8] Maugis D.: Contact, adhesion and rupture of elastic solids. Springer Series in Solid-State Sciences, New York 2000.