TORSION OF ELASTIC TRANSVERSELY ISOTROPIC HALF-SPACE WITH INHOMOGENEOUS COATING

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Summary. An approximate analytical solution of the contact problem of torsion by a flat stamp of an elastic transversely isotropic half-space with inhomogeneous coating is developed. By using the integral transformation technique solution of the problem is reduced to solving an integral equation. Kernel's transform of the integral equation is constructed numerically using method of modeling functions. Based on the established analytical properties, kernel transform is approximated by special analytic expressions. Analytic solutions are constructed for these approximations of integral equations. It is proved that constructed solutions are asymptotically exact for small and big values of dimensionless geometric parameter of the problem. The effect of anisotropy and inhomogeneity are studied separately for each component of the shear modulus. Accuracy of the constructed approximate analytical solutions of the problem is analyzed.

A non-deformable circular indentor with flat base is inflexibly coupled with upper bound Γ of elastic inhomogeneous half-space Ω. The half-space is connected with the cylindrical system of coordinates r, φ, z. Half-space Ω is assumed to be transversely-isotropic. The indentor contacts with the half-space by the surface z=0, $r\le a$. The indentor is effected by torque M, whose axis coincides with the axis z.

Under the influence of this moment the indentor will turn in relation to the axis z on the angle ε, which compels the torsion deformation Ω. Components of shear modulus $G_{z\varphi}$, $G_{r\varphi}$ of the half-space with the depth alternates under the laws:

$$G_{r\varphi}=\begin{cases} G_{r\varphi}(z) & -H\le z\le 0 \\ G_{r\varphi}(-H) & -\infty<z<-H \end{cases},\quad G_{\varphi z}=\begin{cases} G_{\varphi z}(z) & -H\le z\le 0 \\ G_{\varphi z}(-H) & -\infty<z<-H \end{cases}.$$

Outside the indentor Γ is not loaded. Under the made suggestions the boundary conditions of the problem are:

$$z=0,\ \sigma_z=\tau_{rz}=0,\ \begin{cases} \tau_{z\varphi}=0, & r>a \\ u_{\varphi}=r\varepsilon, & r\le a \end{cases}.$$

If $r\to\infty$ or $z\to-\infty$ the stresses vanishing. Suppose that displacement and stresses interface on the boundary of alternation of inhomogeneity law:

$$z=-H,\ \tau_{z\varphi}^{(1)}=\tau_{z\varphi}^{(2)},\ u_{\varphi}^{(1)}=u_{\varphi}^{(2)}.$$

Here index $^{(1)}$ correspond to the coating, while index $^{(2)}$ correspond to substructure - homogeneous half-space.

It is required to determine the law of distribution of contact tangential stresses under the indentor

$$\tau_{z\varphi}\big|_{z=0}=\tau_a(r),\quad r\le a$$

as well as the connection between the applied torque M and the angle of torsion of the indentor.

Using integral transformation technique [1] the solution of the problem is reduced to the solution of integral equation:

$$\int_0^1\tau(\rho)\rho d\rho\int_0^\infty L(u)\mathrm{J}_1(ur\lambda^{-1})\mathrm{J}_1(u\rho\lambda^{-1})du=\lambda G_{\varphi z}(0)r\varepsilon,\ r\le 1.$$

Here $\tau(\rho)=\tau_a(\rho a)$; $\lambda=H/a$ — geometrical parameter of the problem; $L(u)$ — kernel transform of integral equation, $J_1(u)$ — Bessel function of the first kind.

Generally, the value of the kernel's transform $L(u)$ is constructed numerically, using a method of modeling functions [2].

If the transform of the kernel $L(u)$ has the view

$$\Pi_N(u)=\prod_{i=1}^{N}\frac{u^2+A_i^2}{u^2+B_i^2} \tag{1}$$

then, using the bilateral asymptotic method described in work [3], we can construct the solution of the problem:

$$\tau(r)=\frac{4\cdot G_{z\varphi}(0)\varepsilon}{\pi}\left\{\Pi_N^{-1}(0)\frac{r}{\sqrt{1-r^2}}+\sum_{i=1}^{N}C_iZ(r,A_i\lambda^{-1})\right\}, \tag{2}$$

where C_i are determined out of the system of the linear algebraic equations.

$$\sum_{i=1}^{N}C_i\,p\left(\frac{A_i}{\lambda},\frac{B_k}{\lambda}\right)+\frac{1+B_k\lambda^{-1}}{\Pi_N(0)B_k{}^2\lambda^{-2}}=0,\ \ k=1,2,\dots,N$$

and

$$Z(r,A)=\frac{\text{sh } Ar}{r}+\frac{r\,\text{sh } A}{S(1,r)}-Ar\int_r^1\frac{\text{ch } Atdt}{S(t,r)},$$

$$S(t,r)=\sqrt{t^2-r^2}\left(t+\sqrt{t^2-r^2}\right),$$

$$p(A,B)=(A\,\text{ch } A+B\,\text{sh } A)\cdot\left(B^2-A^2\right)^{-1}.$$

Proved that constructed solution is exact if $\lambda\to 0$ or $\lambda\to\infty$.

We study the laws of the inhomogeneity 1-4, in which one component of the shear modulus varies non-monotonically, while the other is constant. Variation of the shear modulus components with depth for laws 1-4 are shown in figure 1.

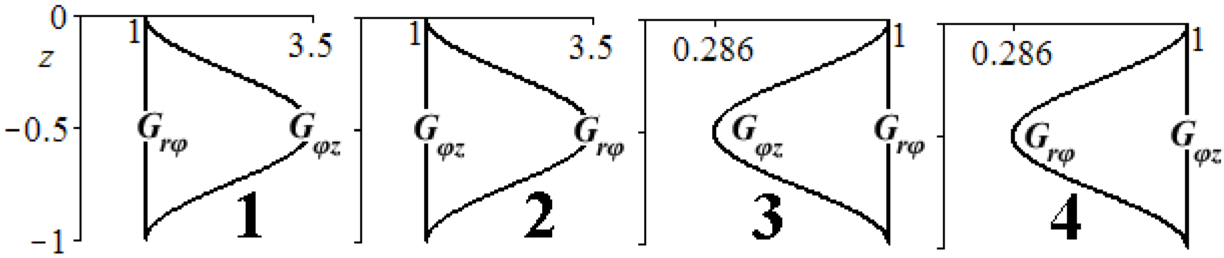

Fig. 1 Variation of the shear modulus components with depth for laws 1-4

To study the effect of anisotropy for each component of the shear modulus, the results are compared with the case of an inhomogeneous isotropic coating.

Using numerical experiments we analyze the accuracy of constructed solutions. Results can also be compared with the results of Grilitskii [4] for transversely-isotropic elastic half-space.

For the first time the problem about torsion by a circular indentor with a flat base of a homogeneous half-space was solved by E. Reissner and H. F. Sagoci [5]. Sneddon I.N. in [1] reduces the problem to a dual integral equation using integral transformation technique. Torsion problem for isotropic two-layered media was studied by Grilitskii V.D. [4]. He gives the solution in the form of power series by δ^{-1}, where δ is a ratio of first layer thickness to the indentor radius. Contact stresses and displacement under the indentor was also found. He also considered the torsion problem for transversely isotropic half-space. Different formulations of the torsion problem for inhomogeneous media were considered in the works [6]-[8].

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