METHODOLOGY FOR MODELING SURFACE AND SUBSURFACE DAMAGE DUE TO CONTACT LOADING

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Summary A general solution is developed for multiple subsurface defects including inclusions and cracks at or near surfaces under contact loading. This solution fully considers the interactions among all the subsurface defects and between them and the surface loading body, thus providing an accurate description of surface deformation and pressure and subsurface elastic field. Based on this solution, a unified methodology could be developed for modeling various damage patterns (including surface fracture-based wear and gradual wear and subsurface cracking and plastic zone expansion), their competitions, and surface evolutions due to them.

INTRODUCTION

Materials are often subject to surface and subsurface damages such as wear, cracking, and plastic deformation. A proper prediction of such damages is of critical importance in material and structural design; however, the prediction is tremendously complicated both experimentally and theoretically by the presence of inhomogeneous inclusions formed in materials during the manufacturing process. An inhomogeneous inclusion has different material properties than the surrounding matrix; it may or may not contain eigenstrain which refers to inelastic strain such as thermal strain, plastic strain, etc. In contrast, a homogenous inclusion has the same material as the matrix but contains eigenstrain.
Eshelby pioneered the work on inhomogeneous inclusions by solving an ellipsoidal inhomogeneous inclusion embedded in an infinite space [1]. Due to mathematical complexity, the interactions among more than two inhomogeneous inclusions have barely been studied. The problems are further complicated by considering inhomogeneous inclusions near surfaces. More difficulties are added by considering surface contact loading or surface deformation. So far, in the literature only few studies that consider contacting loading have been done. Recently, the authors and co-workers have done a series of works on multiple arbitrarily-shaped inhomogeneous inclusions [2-4]. Based on them, this paper presents a unified methodology to model surface and subsurface damage due to contact loading.

METHODOLOGY AND DISCUSSION

Firstly, we study a half space with elastic moduli $\mathbf{C}$ that contains multiple inhomogeneous inclusions Ω_ψ $(\psi = 1, 2, \ldots, n)$ with elastic moduli $\mathbf{C}^\psi$ near surface under prescribed loading. Eshelby's equivalent inclusion method (EIM) states that an inhomogeneous inclusion can be treated as a homogeneous inclusion with initial eigenstrain $\boldsymbol{\varepsilon}^{\mathrm{p}}$ plus equivalent eigenstrain $\boldsymbol{\varepsilon}^*$ to be determined [1]. Using the EIM, the inhomogeneous inclusion problem is converted to a homogenous inclusion problem. The equivalent eigenstrains are introduced to represent material differences of the inhomogeneous inclusions, the interactions among them, and their responses to the initial eigenstrains and surface tractions. The governing equation is given as

$$(\mathbf{C}^\psi \mathbf{C}^{-1} - \mathbf{I})\boldsymbol{\sigma}^* + \mathbf{C}^\psi \boldsymbol{\varepsilon}^* = (\mathbf{I} - \mathbf{C}^\psi \mathbf{C}^{-1})(\boldsymbol{\sigma}^{\mathrm{p}} + \boldsymbol{\sigma}^0) \;, \qquad (\psi = 1, 2, 3, \ldots, n), \tag{1}$$

where $\mathbf{I}$ is a unit matrix, $\boldsymbol{\sigma}^*$ the eigenstress caused by the equivalent eigenstrains $\boldsymbol{\varepsilon}^*$ in all the homogeneous inclusions; $\boldsymbol{\sigma}^{\mathrm{p}}$ the eigenstress caused by the initial eigenstrains $\boldsymbol{\varepsilon}^{\mathrm{p}}$ in all the homogeneous inclusions; and $\boldsymbol{\sigma}^0$ the applied stress due to surface loading. To explicitly express $\boldsymbol{\sigma}^*$ and $\boldsymbol{\sigma}^{\mathrm{p}}$, the solution for multiple homogenous inclusions is needed. A computational domain that contains all the homogenous inclusions is selected and then discretized into small cuboids [**5**]. Accordingly, each homogenous inclusion is approximated by a collection of small cuboids. The eigenstrain in each cuboid is treated as uniform, but overall eigenstrains in each homogenous inclusion remain non-uniform. The solution for a cuboid containing uniform eigenstrain in a half-space was obtained by Chiu **[**6**].** By summing up the contributions from each cuboid that contains uniform eigenstrain, the solution for multiple homogenous inclusions can be obtained. With $\boldsymbol{\sigma}^*$ and $\boldsymbol{\sigma}^{\mathrm{p}}$ as input, Eq. (1) becomes solvable and unknown equivalent eigenstrains $\boldsymbol{\varepsilon}^*$ can be determined iteratively. Once unknowns $\boldsymbol{\varepsilon}^*$ are determined, the entire elastic field of the inhomogeneous inclusions near loaded surfaces is solved.

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Secondly, we study the same half space but subjected to contact loading. The new feature is that the surface deformation of the half space should be considered. The surface deformation is caused by both surface contact loading and the presence of subsurface defects including inhomogeneous inclusions near the surface. There exist interactions between subsurface defects and the surface loading body that applies the surface pressure and friction. The normal external load applied on the loading body is given, but the surface contact area and pressure between the loading body and the half space are not known yet and determined by the following equations:

$$W_{\mathrm{N}} = \iint_{A_c} p(x,y)\mathrm{d}x\mathrm{d}y, \qquad h(x,y) = h^{\mathrm{i}}(x,y) + u_z(x,y) - \delta_z \geq 0, \tag{2}$$

$$h(x,y) = 0, \quad p(x,y) > 0, \quad (x,y) \in A_{\mathrm{c}}; \qquad h(x,y) > 0, \quad p(x,y) = 0, \quad (x,y) \notin A_{\mathrm{c}}, \tag{3}$$

In Eq. (2), the double integration of the normal contact pressure $p(x,y)$ in the contact area A_{c} balances the normal component W_{N} of the external load; $h(x,y)$ describes the gap between two surfaces in contact, $h^{\mathrm{i}}(x,y)$ describes their initial gap; $u_z(x,y)$ represents the composite surface placement; δ is the relative "rigid-body" approach. The composite surface displacement $u_z(x,y)$ is defined as the sum of the elastic deflections of two contacting surfaces. In Eq. (3), the surface gap is positive outside the contact area, but is zero within it.

Using the EIM, the original problem of inhomogeneous inclusions near surfaces in contact can be converted into the new problem of homogenous inclusions near surfaces in contact. The new problem can be decomposed into two sub-problems. One is the homogenous inclusion problem in which equivalent eigenstrains are to be determined. The other is a homogeneous half-space contact problem in which contact pressure is to be determined. Since the two sub-problems are correlated, an algorithm was developed to integrate them. At first, the surface geometry is initialized. Then, a homogenous half-space contact model is used to obtain the surface contact pressure. Next, the inhomogeneous inclusion model is used to determine the equivalent eigenstrains. After that, the surface displacement due to all the equivalent eigenstrains and initial eigenstrains is calculated. If the eigen-displacement converges, the calculation is completed. Otherwise, the surface geometry is updated with the eigen-displacement and the next iteration step begins. The discretized contact model utilizes Eqs. (2)-(3).

Thirdly, a plastic zone in material can be regarded as a homogenous inclusion of the material. Based on the model for inhomogeneous inclusions near surfaces in contact, an elastic-plastic indentation model is further developed. This indentation model can study the plastic deformation of surfaces as well as subsurface plastic zone expansion around inhomogeneous inclusions, which generally cause stress concentration that lead to material yielding. The new feature of the elastic-plastic indentation model is that it can determine material yielding based on von Mises yield criterion and plastic flow rule. Furthermore, a layer of film on a substrate can be modeled as an inhomogeneous inclusion of the same thickness embedded in a half space of the substrate material, resulting in the modeling of film-substrate systems.

This methodology is capable of providing an accurate knowledge of surface pressure and subsurface stress field for a complicated system involving material dissimilarity and surface roughness. It can be used to study gradual wear by incorporating the Archard law, to study fracture-based wear by modeling cracks by eigenstrains, and to study imperfect bonding and film delamination by changing the boundary conditions. More interestingly, the methodology can be applied to study the competition among different damage patterns and track the surface evolution due to them.

CONCLUSIONS

This paper presents a unified methodology that can be used to model various types of surface and subsurface damages. The methodology is based on a general solution for subsurface defects including inclusions and cracks at or near surfaces under contact loading. The solution takes into account the interactions among all the subsurface defects and the interactions between them and the surface loading body, and thus provides an accurate knowledge of surface deformation and pressure and subsurface elastic field.

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