PLANE NONSTATIONARY CONTACT PROBLEMS OF THE IMPACT OF RIGID AND DEFORMABLE STRIKERS ON AN ELASTIC HALF-SPACE

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Summary The plane nonstationary contact problems with moving boundaries for perfectly rigid and deformable strikers and the elastic half-space are studied. The methods and algorithms for the solution based on the use of surface influence functions and the superposition principle are developing. A technique for the analytic investigation of features of contact stresses in the vicinity of the nonstationary moving boundary of a contact region is proposed.

STATEMENT OF THE PROBLEMS

At the initial time is infinitely long striker, bounded by a smooth convex surface given by the equation $z_1 = f(x_1)$, moving at an initial speed V_0, comes into contact with an homogeneous isotropic linearly elastic half-space. Strikers are considered absolutely rigid and elastic bodies, as well as thin cylindrical shell. Initially, the contact occurs along a generator of the boundary surface of the striker, which leads to a flat statement of the problem (Fig. 1).

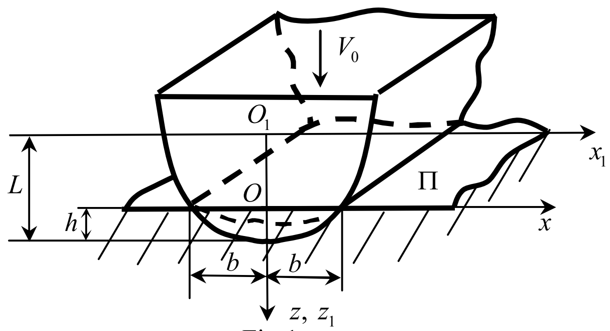

Fig.1

A closed mathematical model for unsteady contact interaction consists of the following equation.

The equations of motion of the striker

$$\mathbf{A}(\mathbf{u}_0) = \rho_0 \ddot{\mathbf{u}}_0 \tag{1}$$

Here and below, quantities with a subscript « 0 » refers to the striker. $\mathbf{A}$ is the matrix of differential operators by the space variables corresponding to the specific model of striker; ρ_0 is the density of material of the striker; $\mathbf{u}_0 = (u_0, 0, w_0)^T$ is the vector of elastic displacements of the striker. From now on, a dot over a symbol denotes a derivative with respect to the time.

The equations of motion of the isotropic linearly elastic half-space

$$c_1^2 \Delta\varphi = \ddot{\varphi}\,,\quad c_2^2 \Delta\psi = \ddot{\psi}\,, \tag{2}$$

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2},\quad c_1^2 = \frac{\lambda + 2\mu}{\rho},\quad c_2^2 = \frac{\mu}{\rho},\quad \mathbf{u} = \mathrm{grad}\varphi + \mathrm{rot}\boldsymbol{\psi},\quad \boldsymbol{\psi} = (0,\psi,0)^T,\quad \mathbf{u} = (u,0,w)^T.$$

Here $\mathbf{u}$ - is the vector of elastic displacements of the half-space; φ, $\boldsymbol{\psi}$ are the scalar and vector potentials of elastic displacements of the half-space; λ, μ, ρ are the Lamer elastic constants and the density of material of the striker.

Boundary condition

In the linear statement of the problem, the contact region is approximately replaced by the segment $[-b(t), b(t)]$, which belongs to the unperturbed free surface of the half-space $z = 0$ (Fig. 1):

$$\begin{gathered} \sigma_{13}(x,t,0) = 0,\ x \in (-\infty,\infty);\ \sigma_{33}(x,t,0) = 0,\ |x| > b(t); \\ \sigma_{33}(x,t,0) = p(x,t),\ w(x,t,0) - w_0(x,t,0) = h(t) + f(x) - L,\ |x| \le b(t); \\ \lim_{r\to\infty}\varphi = \lim_{r\to\infty}\psi = 0,\ r^2 = x^2 + z^2. \end{gathered} \tag{3}$$

Here $\sigma_{13}(x,t,z)$, $\sigma_{33}(x,t,z)$ are the components of the stress tensor; p is the contact pressure; L is the distance from the center of mass O_1 to the frontal point of the striker/

The kinematic relation for determining the half-width of the contact area

$$f(b) = L - h \tag{4}$$

Here h is the penetration depth of the striker as a rigid body.

The equation of motion of the striker as a rigid body

$$m_0 h = R(t) = 2\int_0^{b(t)} p(x,t)\,dx \tag{5}$$

Here m_0 is the mass per unit length of the striker.

Initial conditions

$$\varphi\big|_{t=0} = \varphi\big|_{t=0} = \psi\big|_{t=0} = \psi\big|_{t=0} = 0,\ h\big|_{\tau=0} = 0,\ h\big|_{t=0} = V_0,\ \mathbf{u}_0\big|_{t=0} = \mathbf{0},\ u_0\big|_{t=0} = -V_0\sin\vartheta,\ w_0\big|_{t=0} = V_0\cos\vartheta,\ \vartheta = \mathrm{arctg}\frac{x}{f(x)}. \tag{6}$$

SYSTEM OF RESOLVING EQUATIONS

Superposition principle and the boundary conditions (3) lead to the integral equation

$$\int_0^{\tau}\int_{-b(t)}^{b(t)} G(x-\xi,t-\tau)\,p(\xi,\tau)\,d\tau d\xi - \int_0^{\tau}\int_{-b(t)}^{b(t)} G_0(x-\xi,t-\tau)\,p(\xi,\tau)\,d\tau d\xi = h(t) + f(x) - L \tag{7}$$

Here $G(x,t)$, $G_0(x,t)$ are the surface influence functions of the half-space and the striker.

The equation (7) together with the kinematic condition (4) and the equation of motion of the striker as a rigid body (5) written in integral form

$$h = V_0 t + \frac{2}{m_0}\int_0^{\tau}\int_0^{b(t)} (t-\tau)\,p(\xi,\tau)\,d\tau d\xi \tag{8}$$

are a closed system of resolving equations.

METHOD OF SOLUTION

The numerical-analytical algorithm based on the method of quadrature with the separating of the singular component of the kernels of integral operators in (7) for the solution of the system of equations (5), (7) and (8) has been developed. The obtained numerical results show that the contact stresses increase significantly in the vicinity of the boundaries of the interaction region. Therefore, in the question of clarifying the existence and nature of the features of contact stresses. For this purpose the analytical technique based on the reduction of two-dimensional singular integral equation (7) to the system of one dimensional integral equations has been developed.

$$\int_{-1}^{1} \frac{g(\psi)}{\psi^2 - c_R^2} Q(\psi;x,t)\,d\psi = w_0(x.t),\quad \int_0^{\beta(\psi;x,t)} q(r,\psi;x,t)\,dr = Q(\psi;x,t), \tag{9}$$

$$\xi = x + r\sin\varphi,\ \ \tau = t - r\cos\varphi,\ \ \psi = \mathrm{tg}\varphi,\ \ q(r,\psi;x,t) = \sigma(r,\mathrm{arctg}\varphi;x,t),\ \ \sigma(r,\varphi;x,t) = p(x + r\sin\varphi, t - r\cos\varphi).$$

Here c_R is the velocity of Rayleigh waves; $r = \beta(\psi;x,t)$ is the equation, which determines the position of the boundary of contact area in the coordinate system O_2, r, ψ ; $O_2 = (x,t)$.

The first of equations (9) reduces to an equivalent pair of Riemann problem [1-2]. Its solution, and thus the decision of the first equation, can be found analytically. Analysis of the second integral equation shows that to ensure the uniqueness of its solution must possess a power singularity. In this case, it turns into a generalized equation of Abel. His solution is investigated using the technique of fractional integro-differentiation [3]. It is shown that the solution has the following structure

$$q(r,\psi;x,t) = \frac{q(r,\psi;x,t)}{\left[\beta(\psi;x,t) - r\right]^{\alpha}} \tag{10}$$

Here $q(r,\psi;x,t)$ is the function bounded on the boundary of the contact area.

В результате исследования доказано существование особенности контактных напряжений в окрестности границ области взаимодействия в нестационарных контактных задачах. Её характер существенно зависит от закона распространения границ области контакта по поверхности упругого полупространства. The study proved the existence of power singularity of contact stresses in the vicinity of the boundaries of interaction in unsteady contact problems. Its character depends on the law of propagation boundaries of the contact area on the surface of the elastic half-space. It should be noted that in the case of transition to a static assignment of tasks the order of singularity α is zero. This is consistent with the known solutions of static contact problems.

References

[1] Muskhelishvili, N.I.. Some Basic Problems of the Mathematical Theory of Elasticity. Springer, 1977.

[2] F.D. Gakhov, Boundary value problems, Moscow: Nauka, 1977.

[3] S.G. Samko, A.A. Kilbas, O.I. Marichev: Integrals and Derivatives of Fractional Order and Some of Their Applications, Nauka i Tekhnika, Minsk, 1987.