METHOD OF REDUCTION OF DIMENSIONALITY IN CONTACT MECHANICS

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Summary: In 2006, Geike and Popov have shown that there is a wide class of contacts between three-dimensional bodes which can be mapped either exactly or without loss of essential information to one-dimensional systems (elastic or visco-elastic foundations). In 2011, Heß proved many of the mapping theorems and has shown that the *exact* mapping is always possible for *any* axis-symmetric body, both with and without adhesion. The present paper presents the basic ideas and applications of the method of reduction of dimensionality for simulation of normal and tangential contacts of elastic and visco-elastic materials with adhesion.

Tribological simulation with the method of reduction of dimensionality

Computer simulations are now an important part of any technological development process. Modern structural mechanics, fluid dynamics, acoustics and electrical engineering, as well as almost any other technological field cannot be imagined without numerical simulations. Tribology is one of the last fields where computer simulations has not played any important role up until now. The reason is that in tribological phenomena, all scales up to the nanoscale have to be taken into account. In 2006, a new simulation method was proposed, which allows for the first time the simulation of numerically technologically relevant systems and phenomena [1]. It was shown that under some restrictions, which are, however, fulfilled in many tribological problems, it is possible to map three-dimensional systems onto equivalent one-dimensional Winkler foundations. This reduction includes not only the reduction of a three-dimensional system to a one-dimensional one, but also the independency of the resulting degrees of freedom. At the first glance, such reduction seems to be impossible. However, rigorous proofs have been given during 2011 for wide classes of surface profiles and material rheologies [2]. There is still no formal proof for randomly rough surfaces; however, the validity of the method for this class of surface profiles has been verified by comparison with sophisticated exact three-dimensional simulations [3], [4].

The method of reduction of dimensionality means a *huge* reduction of computational time for simulation of contact and friction between rough surfaces accounting for complicated rheology and adhesion. Because of independence of single "springs" of equivalent elastic foundations, it is ideal for parallel calculation on graphic cards. The method allows for the first time for microscopic contact mechanics and simulation of macroscopic system dynamics to be combined without determining the "law of friction" as an intermediate step.

Basic idea of the reduction method

The fundamental idea of the method of reduction of dimensionality is the following. If a round indenter is pressed into the surface of an elastic continuum, then the stiffness k of the contact is *proportional to its diameter* D:

$$k = DE^*, \tag{1}$$

where $E^* = \left(\left(1-\nu_1^2\right)/E_1 + \left(1-\nu_2^2\right)/E_2\right)^{-1}$ denotes the effective elastic modulus. E_1, E_2, ν_1 and ν_2 are the Young's moduli and Poisson's ratios of the contacting bodies. *This property* can be reproduced using a one-dimensional *elastic foundation* (Fig. 1a).

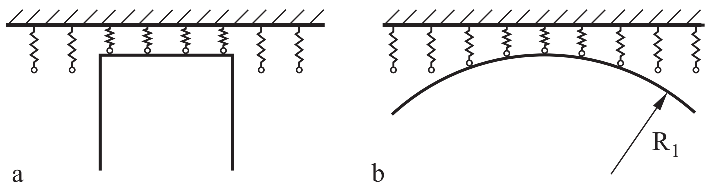

Fig. 1 One-dimensional elastic foundation in contact with a "cylindrical" indenter (a) and a "sphere" (b).

In order to fulfill Equation (1), the stiffness per unit length must be chosen as E^*. Every individual spring must have the stiffness

$$\Delta k = E^* \Delta x . \tag{2}$$

If a "sphere" with the radius R_1 is brought into contact with the elastic foundation (penetration depth d), seen in Fig. 1b, then the following contact quantities result: the contact radius is equal to

$$a = \sqrt{2R_1 d} \tag{3}$$

and the normal force is

$$F_N(d) = \frac{4\sqrt{2}E^*}{3}\sqrt{R_1 d^3}\ . \tag{4}$$

If we choose a radius

$$R_1 = R/2\,, \tag{5}$$

then the Equations (3) and (4) coincide exactly with Hertzian theory.
Thus, the contact of a rotationally symmetric three-dimensional rigid body with an elastic continuum can be represented by the contact of a corresponding one-dimensional cross-section having half the radius of curvature with an elastic foundation having the stiffness per unit length of E^*. This rule is *exactly* correct for *every* cylindrical indenter and for *every* parabolic body with an *arbitrary* radius of curvature. It was further shown in [2] that the method of reduction of dimensionality is exact valid for any body of revolution with an arbitrary polynomial profile.

Reduction method for a multi-contact problem
In order to cross over to a contact between bodies with rough surfaces, a rule for the production of a one-dimensional profile which is equivalent to the three-dimensional body in a contact mechanical sense must be formulated (Fig. 2).

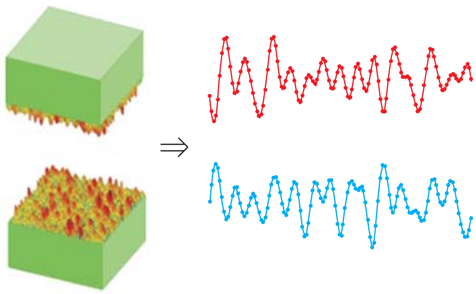

Fig.2 Replacing two three-dimensional bodies with two equivalent one-dimensional “rough lines.”

As shown in [3], in order to produce a one-dimensional system with the same contact properties as the three-dimensional system, the one-dimensional power spectrum $C_{1\mathrm{D}}(q)$ must be used according to the rule

$$C_{1\mathrm{D}}(q) = \pi q C_{2\mathrm{D}}(q)\,, \tag{6}$$

where $C_{2\mathrm{D}}(q)$ is the power spectrum of the initial 2D surface.

Reduction method for a visco-elastic contacts and adhesion
As shown in [2] and [5], the method of reduction of dimensionality can also be used for *exact* theating contact problems with *arbitrary* linear rheology, with adhesion, tangential contacts with friction and for calculating the stress distributions in the initial three-dimensional problem based on the 1D model.

Applications
In the presentation, several applications will be shown, including simulation of nano-robots and actuators based on the stick-slip-principle.

CONCLUSIONS

A breakthrough method for multi-scale simulation of contact and friction between rough surfaces is presented and illustrated with applications in contact mechanics of stick-slip motors and actuators.

References

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