MULTIHARMONIC BALANCE ANALYSIS OF A FRICTION OSCILLATOR INCLUDING A BOLTED JOINT

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Summary An usual approach to investigate nonlinear dynamical systems in the frequency domain is the application of the Harmonic Balance Method (HBM), assuming that a harmonic excitation of the system leads to a harmonic response. However, for systems where the steady state response is not just harmonic but periodic or for systems which are excited periodically, this assumption does not longer lead to satisfying results. Therefore, the Multi Harmonic Balance Method (MHBM) is utilized and embedded into the Finite Element Method using Zero Thickness Elements.

The differential equations describing a nonlinear system may be written as

$$\boldsymbol{M}\ddot{\boldsymbol{u}}(t) + \boldsymbol{D}\dot{\boldsymbol{u}}(t) + \boldsymbol{K}\boldsymbol{u}(t) + \boldsymbol{f}_{nl}(\dot{\boldsymbol{u}}, \boldsymbol{u}, t) = \boldsymbol{f}_e(t)\,. \tag{1}$$

In the framework of the MHBM, the single-harmonic ansatz of the HBM describing the response-displacements $\boldsymbol{u}(t)$ of the system is extended to

$$\boldsymbol{u}(t) = \sum_{k=0}^{n_h} \left(\boldsymbol{U}_k \cos(k\omega t)\right) = \frac{1}{2}\sum_{k=0}^{n_h} \left(\boldsymbol{U}_k \left(e^{i\cdot k\omega t} + e^{-i\cdot k\omega t}\right)\right)\ , \tag{2}$$

considering n_h harmonic parts. When the nonlinear forces $\boldsymbol{f}_{nl}$ are approximated by a FOURIER-series, the FOURIER-coefficients $\boldsymbol{a}_k$ and $\boldsymbol{b}_k$ depend on every harmonic $\boldsymbol{U}_i$ of the response displacement,

$$\boldsymbol{a}_k = \boldsymbol{a}_k(\boldsymbol{U}_0, \boldsymbol{U}_1,\ \ldots,\ \boldsymbol{U}_k,\ \ldots,\ \boldsymbol{U}_{n_h}) \quad \text{and} \quad \boldsymbol{b}_k = \boldsymbol{b}_k\left(\boldsymbol{U}_0, \boldsymbol{U}_1,\ \ldots,\ \boldsymbol{U}_k,\ \ldots,\ \boldsymbol{U}_{n_h}\right). \tag{3}$$

Therefore, it is not possible to formulate equivalent coefficients as it is done with the HBM. Since the analytical calculation of the FOURIER-coefficients is not practicable, see e.g. [1], the so called Alternating Frequency Time Domain Method (AFT), see [2], is applied leading to an iterative solution scheme switching from frequency domain to time domain and vice versa. For doing so, a NEWTON-RAPHSON-type predictor-corrector algorithm is employed. The required partial derivatives of the nonlinear forces with respect to the response displacements can be calculated analytically in the time domain, see [3] or [4]. The MHBM/AFT is used to calculate frequency response functions (FRF) of a friction oscillator which consists of two masses, a bending spring and a friction type nonlinearity in form of a bolted joint, see figure 1.

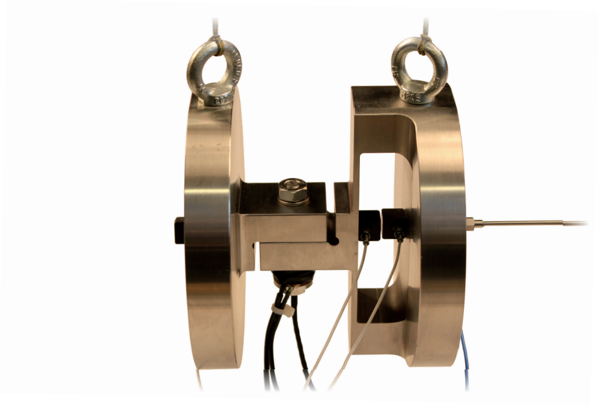

Figure 1. Friction oscillator

It is possible to find a very simple 3-DOF model of this oscillator, see e.g. [5], leading to numerical results correlating very well with measured FRF, see e.g. [6].
Alternatively, this structure can be modeled with the Finite Elemenet Method using Zero Thickness Elements, see e.g. [7] or [8]. For the contacting bodies small deformations and linear elastic material behaviour are assumed. Since the contact area is known and only small relative displacements occur, the suggested contact element is well suited for the present problem. The following equations are arranged considering the local contact stresses $\boldsymbol{t} = [t_n\ \boldsymbol{t}_t]^T$ and relative displacements $\boldsymbol{g} = [g_n\ \boldsymbol{g}_t]^T$, respectively. The constitutive law in normal direction for the case of contact is linear elastic with the stiffness coefficient c_n and the normal relative displacement g_n. For the case of separation, the local normal stress t_n in the k-th time step is set to zero:

$$t_n^{(k)} = \begin{cases} 0 & \text{separation} \\ c_n g_n & \text{contact} \end{cases}\,. \tag{4}$$

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Applying a radial return mapping algorithm, the local tangential contact stresses $\boldsymbol{t}_t$ within the contact plane may be written as

$$\boldsymbol{t}_t^{(k)} = \begin{cases} 0 & \text{separation} \\ {}^{tr}\boldsymbol{t}_t^{(k)} & \text{stick} \\ {}^{m}t_t^{(k)} \dfrac{{}^{tr}\boldsymbol{t}_t^{(k)}}{\| {}^{tr}\boldsymbol{t}_t^{(k)} \|} & \text{slip} \end{cases} , \tag{5}$$

where the maximal tangential stress is defined as

$${}^{m}t_t^{(k)} = -\mu t_n^{(k)} \tag{6}$$

with the friction coefficient μ. The tangential trial-stresses are

$${}^{tr}\boldsymbol{t}_t^{(k)} = c_t \overline{\boldsymbol{m}} \left(\boldsymbol{g}_t^{(k)} - \boldsymbol{g}_t^{(k-1)} \right) + \boldsymbol{t}_t^{(k-1)} , \tag{7}$$

where c_t is the tangential stiffness coefficient and $\overline{\boldsymbol{m}}$ is the contravariant metric, transforming the covariant displacements into contravariant coordinates.

For the MHBM procedure the partial derivatives of the contact stresses with respect to the relative displacements in the frequency domain,

$$\begin{bmatrix} \dfrac{\partial \Re\{\boldsymbol{T}^{(q)}\}}{\partial \Re\{\boldsymbol{G}^{(p)}\}} & \dfrac{\partial \Re\{\boldsymbol{T}^{(q)}\}}{\partial \Im\{\boldsymbol{G}^{(p)}\}} \\ \dfrac{\partial \Im\{\boldsymbol{T}^{(q)}\}}{\partial \Re\{\boldsymbol{G}^{(p)}\}} & \dfrac{\partial \Im\{\boldsymbol{T}^{(q)}\}}{\partial \Im\{\boldsymbol{G}^{(p)}\}} \end{bmatrix} , \tag{8}$$

shall be computed. As mentioned above, these partial derivatives are calculated in the time domain and are then transformed back into the frequency domain using the discrete FOURIER-transform $\mathcal{F}$. A consistent derivation of the constitutive laws (4) and (5), analogously to e.g. [9] and [4], is possible. As an example, the partial derivatives of the tangential contact sresses with respect to the real parts of the normal relative displacements can be written as

$$\frac{\partial \boldsymbol{T}_t^{(q)}}{\partial \Re\{G_n^{(p)}\}} = \frac{\partial \Re\{\boldsymbol{T}_t^{(q)}\}}{\partial \Re\{G_n^{(p)}\}} + \frac{\partial \Im\{\boldsymbol{T}_t^{(q)}\}}{\partial \Re\{G_n^{(p)}\}} = \mathcal{F}^{(q,k)} \left\{ \frac{\partial \boldsymbol{t}_t^{(k)}}{\partial \Re\{G_n^{(p)}\}} \right\} , \tag{9}$$

with

$$\frac{\partial \boldsymbol{t}_t^{(k)}}{\partial \Re\{G_n^{(p)}\}} = \begin{cases} \boldsymbol{0} \\ \dfrac{\partial \boldsymbol{t}_t^{(k-1)}}{\partial \Re\{G_n^{(p)}\}} \\ \dfrac{1}{\| {}^{tr}\boldsymbol{t}_t^{(k)} \|} \left({}^{m}t_t^{(k)} \dfrac{\partial \boldsymbol{t}_t^{(k-1)}}{\partial \Re\{G_n^{(p)}\}} - \mu c_n \cos\left(kp \dfrac{2\pi}{N} \right) {}^{tr}\boldsymbol{t}_t^{(k)} \right) - \dfrac{{}^{m}t_t^{(k)}\, {}^{tr}\boldsymbol{t}_t^{(k)}}{\| {}^{tr}\boldsymbol{t}_t^{(k)} \|^3} \left({}^{tr}\boldsymbol{t}_t^{(k)}\, \overline{\boldsymbol{m}}\, \dfrac{\partial \boldsymbol{t}_t^{(k-1)}}{\partial \Re\{G_n^{(p)}\}} \right) \end{cases} \tag{10}$$

for the three cases separation, stick and slip respectively. Using the partial derivatives computed this way, the FRF of the system can be calculated. A parameter identification, e.g. based on a least squares algorithm, can be applied using measured FRF to identify the three unknown contact parameters c_n, c_t and μ.

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