A UNIFIED ANALYSIS OF A MICRO-BEAM, DROPLET AND CNT RING ADHERED ON A SUBSTRATE

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Summary Numerous phenomena of nature, which capture similar characteristics and obey the same governing equation, can be ascribed to a unified framework. Here we developed a general method to analytically tackle a kind of movable boundary problem from the viewpoint of energy variation. Having grouped the adhesion of a micro-beam, droplet and carbon nanotube (CNT) ring on a substrate into one framework, we used the developed line of reasoning to investigate the adhesion behaviors of these systems. Based upon the derived governing equations and transversality conditions, the explicit solutions of the critical parameters and morphologies for the three systems are successfully obtained, and then the parameter analogies and common characteristics of them are thoroughly addressed. Our analyses can be compared with the concept of energy release rate, which provide a new approach for exploring the mechanism of different systems with similarities as well as for understanding the unity of nature.

INTRODUCTION

Numerously similar phenomena have been miraculously created in nature, spanning from nano-, micro-, and meso-scales to macro-scales. Here we will focus on the similarities in the adhesion of materials and devices at micro- and nano-scales, which are caused by van der Waals force, Casimir force, capillary force or some other interaction forces. Among others, the adhesion of a droplet, micro-beam and carbon nanotube (CNT) ring on a solid is of great value for both theoretical and practical aspects. In these low-dimensional systems with considerable surface to volume ratio, surface tension or interfacial energy dominates over the volume force as their scales reduce to micro/nano-meters, and this feature leads to many novel behaviors distinct from those of the macroscopic systems.

Much effort has already been devoted for the adhesion similarities of a droplet, slender rod and CNT on a solid. It should be stressed that although belonging to different systems, the above three models capture some common characteristics: (1) Their shapes are essentially the equilibrium states resulting from the competition between surface energy and volume energy; (2) A critical point exists between two different domains which should be determined in calculation, and this point can be considered as a movable boundary. To the best of our knowledge, there is still a lack of detailed deduction from the potential energy to the governing equation on the given models. It is noteworthy that the analytical expressions for the shapes of a droplet and a CNT ring adhered on a solid substrate remain open questions to date. In this study, we constructed a general framework to deal with the movable boundary problems. We then applied the developed method to study the adhesion of a micro-beam, droplet, and CNT ring on a substrate. Through the transversality conditions and governing equations originating from the energy variation, we analytically solved the critical adhesion length and deflection of a micro-beam, the morphology of a droplet, and the configuration of a CNT contacting with a substrate. Finally, the parameter analogies of the three models were addressed and a lengthy discussion was presented.

THE UNIFIED ANALYSIS FRAMEWORK

We consider a generalized elastic system represented as a continuous and smooth curve, where part of the curve is adhered by interfacial forces. As schematized by the anti-clockwise arc length s in Fig. 1(a), the total length of the curve is L. The functional of the total potential energy of the system is written as:

$$\Pi\left[y(s,a)\right]=\int_0^a F\left[s,y(s,a),y'(s,a),y''(s,a)\right]\mathrm{d}s-\int_a^L W\mathrm{d}s\,, \tag{1}$$

where the prime represents the derivative with respect to the arc length, and W is the work of adhesion. By using the principle of least potential energy and considering the movable boundary, one can get the governing differential equation, i.e. the Euler-Poisson equation:

$$F_y-\frac{\partial}{\partial s}F_{y'}+\frac{\partial^2}{\partial s^2}F_{y''}=0\,, \tag{2}$$

and the arbitrariness of variation about the point a leads to the transversality condition

$$W=\left[y'F_{y'}+y''F_{y''}-y'\frac{\mathrm{d}F_{y''}}{\mathrm{d}s}-F\right]_{s=a}. \tag{3}$$

This thought casts a new light on solving the problem with movable boundaries. We then use it to analyze some practical topics, and the adhesion of a micro-beam, a droplet and a CNT ring on a substrate (shown in Fig. 1) is selected as a study case.

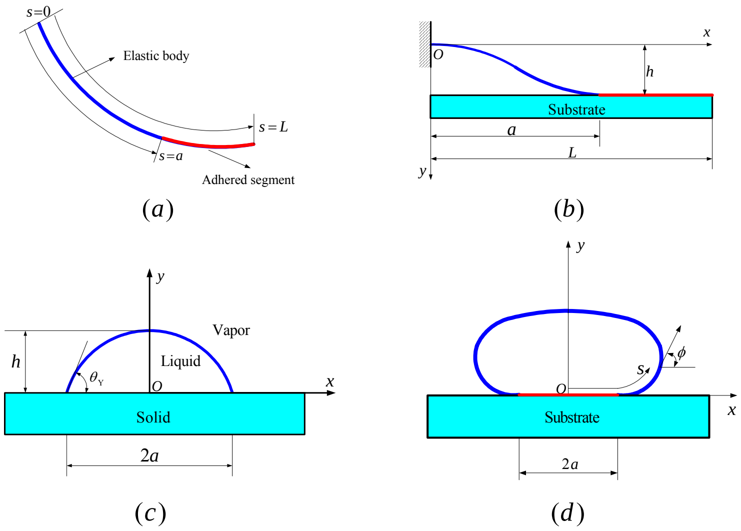

Fig. 1 Adhesion models

RESULTS & DISCUSSION

According to Eqs. (1–3), the comparisons on the three models are as follows:

Iterms	Micro-beam	Droplet	CNT ring
Bending stiffness	EI	γ	EI
Characteristic length	$\sqrt{\gamma/(\rho g)}$	$\sqrt{EI/P}$	$\sqrt{EI/P}$
Interfacial energy	W	$\gamma_{SV}-\gamma_{SL}$	W
Elasto-interface ratio	$\sqrt{EI/W}$	θ_Y	$\sqrt{EI/W}$
Curvature	w''	$y''\left(1+y'^2\right)^{-3/2}$	$\dot{\phi}$
Elastic energy	$\frac{1}{2}EI\int_0^a w''^2\mathrm{d}x$	$\gamma\int_0^a\sqrt{1+y'^2}\,\mathrm{d}x$	$\frac{1}{2}EI\int_a^{\pi R}\dot{\phi}^2\mathrm{d}x$
Euler-Poisson equation	$w^{(4)}=0$	$\gamma y''\left(1+y'^2\right)^{-3/2}=\rho g y-\lambda$	$EI\ddot{\phi}-\lambda_1\sin\phi=0$
Analytical solution	$w=\dfrac{3hax^2-2hx^3}{a^3}$	$\begin{cases}\kappa x=2\left[E(k,\theta)-E(k,\theta_0)\right]-\left[F(k,\theta)-F(k,\theta_0)\right]\\ \kappa y=c_0\kappa^{-1}-2k\cos\theta\end{cases}$	$\begin{cases}\alpha x=\alpha a-F(k,\theta)+2E(k,\theta)\\ \alpha y=2k(1-\cos\theta)\end{cases}$
Transversality condition	$W=\frac{1}{2}EIw''(a)^2$	$\gamma_{SV}-\gamma_{SL}=\gamma\cos\theta_Y$	$W=\frac{1}{2}EI\dot{\phi}(a)^2$
Conservation condition	Small deformation	Mass conservation	Inextensibility

CONCLUSIONS

In this study, we proposed a unified analysis framework for the adhesion of an elastic system with movable boundaries. According to the principle of least potential energy, we derived the governing equation and the transversality condition at the moving bound. We pointed out that the adhesion of a micro-beam, droplet and CNT ring on a substrate can be grouped into this framework, and we put to use our developed method to examine the adhesion behaviors of these systems. The analytical expressions for the detachment length and deflection of the beam, for the Young's equation and morphology of the droplet, and for the contact width and configuration of a CNT ring stuck on a solid substrate can all be derived in consideration of the transversality conditions. These analyses open a new avenue to explore the mechanism of different systems with similarities and the unity of nature in depth. The calculated results are beneficial for the design of nano-structured materials, and cast a light on enhancing their mechanical, chemical, optical and electronic properties.