

A NEW ALGORITHM FOR COLLISION FORCE BETWEEN TWO CABLE-CONNECTED SHIPS

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Summary During skin-to-skin connected replenishment collision between ships excited by wind and wave may occur since some cables might become slack while others still remain tensional. Collision is modelled based on multibody dynamics with unilateral contacts and classical marine hydrodynamics. Computer simulation on maximum collision force conforms to relevant experiments as a whole, which proves that the new algorithm for maximum collision force is available for the design to buffers.

INTRODUCTION

During skin-to-skin connected replenishment at sea two or three ships which are usually connected together with several cables are constantly excited by wind and wave. Collision between the vessels may occur if some cables become slack and the ships are easy to damage when collision force is large enough. So it is necessary for fender to be assigned to arrange on the ships. To evaluate rationally maximum collision force is the base for the design to the rubber buffers. Because of the complexity of skin-to-skin connected replenishment system, collision force usually depends largely on experiment results in actual design. However, experimental study is usually inadequate and expensive. An appropriate algorithm for maximum collision force is desirable. Because some cables might become slack while others still remain tensional, cables and unilateral contacts are two unilateral constraints that floating cable-connected ships in a fluid environment possess. So multibody dynamics with unilateral constraint and stochastic ship dynamics are employed to model the collision. Fluid-structure interaction is dealt with additional water mass taken into account and the hydrodynamic interaction effect between the vessels is ignored. With the most unfavorable conditions considered, vessels are modelled to be rigid and the maximum collision force expression is deduced for the vessels without anti-impact protection device. Relevant numerical simulation is presented to validate the rationality of the new algorithm. In order to improve greatly the efficiency, during simulation process the identification of practical constraints for non-smooth system is changed into an extremum problem and solved with optimization theory. Collision processes can be separated into two phases: a free-motion phase and a contact phase. The free-motion phase is dealt with classical ship dynamics and the contact phase is dealt with stochastic multibody dynamics with unilateral constraint.

EQUATION OF MOTION FOR FREE-MOTION PHASE

$\zeta(\mathbf{r}, t)$ is supposed to be elevation of the surface of sea and is a random process, which is ergodic as regards both space and time statistics. As its corresponding single direction long peak spectrum, Pierson–Moskowitz spectrum $S_{\zeta\zeta}^*(\omega)$ is applied. So in this paper wave load is characterised by the directional wave spectrum recommended by ITTC and ISSC, which has a cosine power spreading function form. The Monte Carlo method from Spanos and Zeldin (1998) suggested that a stochastic process can be simulated by generating only N uniformly distributed random variables. Therefore wave function in time domain $\zeta(t)$ can be approximately expressed in the form of the sum of many cosine waves with different cycles and different stochastic primary phases.

Based on the equations of motion for a single ship at sea investigated by Arnold et al. (2004), for two vessels connected together by six cables, an analogous expression for each ship can be established as follow:

$$M_l \dot{V}_l + \lambda_l \sum_{k=1}^6 T_k = F_{stal} + F_{dynl}, \quad I_l \dot{\Omega}_l + \Omega_l \times I_l \Omega_l + \lambda_l \sum_{k=1}^6 M_{Tk} = M_{stal} + M_{dynl} \quad l = 1, 2$$

$V_l = [\dot{\xi}_{l1}, \dot{\xi}_{l2}, \dot{\xi}_{l3}]^T$, $\Omega_l = [\dot{\xi}_{l4}, \dot{\xi}_{l5}, \dot{\xi}_{l6}]^T$, in which ξ_{lj} indicates the rigid body displacement component of two ships, M_l is each ship and corresponding additional water mass matrix and I_l is each ship moments of inertia. Respectively, F_{stal} and M_{stal} describe the hydrostatic forces and moments. F_{dynl} and M_{dynl} represent the dynamical forces and moments, respectively. These forces, moments and additional water mass achieved according to experiential formula are calculated on the basis of classical marine hydrodynamics from Newman (1977). T_k and M_{Tk} describe the force and moment that are caused by tensions of cables.

SPATIAL CONTACT KINEMATICS AND DYNAMICS OF TWO SHIPS

To determine the positions of the impact points and calculate the length of the cables, coordinate system B and K , are fixed respectively fixed on ship 1 and ship 2. Their cross sections are supposed to be trapezoidal and circular. The

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position of any point on the surfaces of the ships can be expressed by their coordinates (X_{11}, X_{12}, X_{13}) and (x_{21}, x_{22}, x_{23}) , respectively. Experimental evidence shows that impacts may occur at fore, mid-ship and stern. The buffer may therefore be installed at fore, mid-ship and stern. It can be assumed that some symmetry exists in the impact, which must be caused by the special way that the ships are tied together. From the work of Pfeiffer and Glocker (1996), the position of impact points can be determined according to the expression $n^T_1 s_2 = 0$ and the condition that the distance between any two points on the surface of the two ships equal to zero. And With the distance vector of two points at the surface of the two ships and the original length of the cables, the elongation of cables can be determined. The model set up in this paper can be considered as a complementarity-slackness problem as studied by Brogliato (1999). Let constraints $\lambda = \begin{pmatrix} \lambda_N \\ \lambda_L \end{pmatrix}$ and the relative kinematical magnitudes between two ships $f(q, t) = \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = \begin{pmatrix} g_N \\ \Delta L \end{pmatrix}$,

Complementarity-slackness equations can be rewritten as

$$\begin{aligned} M\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) - \frac{\partial \mathbf{f}^T}{\partial \mathbf{q}} \boldsymbol{\lambda} &= \mathbf{P}(t) \\ \mathbf{f}_I(\mathbf{q}, t) &\geq 0 \quad \boldsymbol{\lambda}_N \geq 0 \quad \boldsymbol{\lambda}_N^T \mathbf{f}_I = 0 \\ \mathbf{f}_2(\mathbf{q}, t) &\geq 0 \quad \boldsymbol{\lambda}_L \geq 0 \quad \boldsymbol{\lambda}_L^T (\mathbf{L}_0 - \mathbf{f}_2) = 0 \end{aligned}$$

Based on the work of Sinitsyn (1990), this problem can be

transformed into an optimization problem to be solved by optimization algorithms according to generalized Gaussian least action principle.

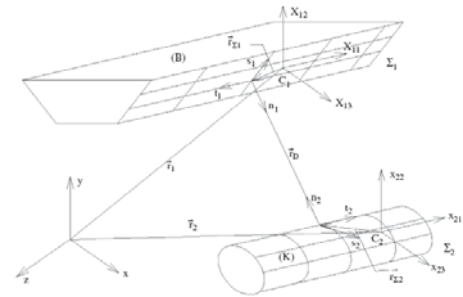


Fig.1. Contact geometry of two ships.

The objective function can be written as $\min_{\lambda \geq 0} G(\lambda) = \frac{1}{2} \lambda^T (\nabla_q \mathbf{f}(\mathbf{q}))^T M^{-1} \nabla_q \mathbf{f}(\mathbf{q}) \lambda + \lambda^T [(\nabla_q \mathbf{f}(\mathbf{q}))^T \mathbf{z} + \frac{d}{dt} (\nabla_q \mathbf{f}(\mathbf{q}))^T \dot{\mathbf{q}}]$

where $\mathbf{z} = \mathbf{M}^{-1}(-\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{P}(t))$

CALCULATION OF MAXIMUM COLLISION FORCE AND NUMERICAL SIMULATION

It is assumed that collision takes place in an infinitesimal short time and without any change of position, orientation and all non-impulsive forces. The relative velocity is assumed to follow the Newton law. By the integration of the dynamics equation over the impact interval the impulse of impact points can be deduced as follow:

$$\Lambda_{Nmax} = \max \left\{ -\mathbf{G}_{NN}^{-1} [(\mathbf{E} + \bar{\varepsilon}_N) \dot{\mathbf{g}}_{NA} + \mathbf{G}_{NL} \tilde{\mathbf{L}}_L] \right\} \text{ where } \mathbf{G}_{NN} = \mathbf{W}_N^T \mathbf{M}^{-1} (\nabla_q \mathbf{f}_1(\mathbf{q}))^T, \mathbf{G}_{NL} = \mathbf{W}_N^T \mathbf{M}^{-1} (\nabla_q \mathbf{f}_2(\mathbf{q}))^T,$$

and $W_N^T = J_{k1}^T \mathbf{n}_1 + J_{k2}^T \mathbf{n}_2$, in which, J_{k1} , J_{k2} are the Jacobians of translation of any two points k_1 and k_2 on the surface of the two ships. The diagonal matrix $\bar{\varepsilon}_N = \text{diag} \{ \varepsilon_{Ni} \}$, which consists of three coefficients of restitution. $\mathbf{g}_{NA}$ is relative velocities in normal direction at the beginning of collision and $\tilde{\Lambda}_f$ describes impulses in cables.

If τ_m is mean of the impact time, the maximum collision force at the contact points $F_l = \Lambda_{Nmax} / \tau_m$.

Monte Carlo simulation from Spanos and Zeldin (1998) is applied to model Pierson-Moskowitz spectrum. Maximum, mean value and standard deviation of collision force are respectively calculated when collision occurs at fore, mid-ship and stern. Numerical maximum collision force at fore and at sternward are a little larger than corresponding experimental results while maximum at mid-ship is much less than the experimental value.

CONCLUSIONS

Fortunately, only maximum collision force is needed to design all the buffers on a ship and the theoretical maximum is a little larger than the experimental one. So the new algorithm is available and it is on the safe side for the design.

References

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