FREQUENCY ANALYSIS OF KNOT MASS PARTICLES IN OSCILLATORY SPHERICAL NET MODEL OF MOUSE *ZONA PELLUCIDA*

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Summary: The zona pellucida (ZP), spherical acellular mantel that surrounds mammalian oocyte dynamically changes its mechanical properties during fertilization. By using discrete oscillatory spherical net model of ZP, previously described by system of differential equations and their solutions, a trajectory analysis of knot mass particles are presented. According to the model, knot mass particles can move in three orthogonal directions and each of these directions is multifrequency vibration asynchronous. Resultant of nonlinear dynamics of knot mass particles is space trajectory in the form of the generalized Lissajus curves. Favorable and unfavorable trajectory of knot mass particle motions are discuss in the context of successful fertilization.

INTRODUCTION

ZP, a spherical acellular mantel that surrounds mammalian oocyte, is a very important structure in process of oogenesis, fertilization, and preimplantation development. ***ZP*** elasticity differs before and after fertilization. Upon fertilization *ZP* undergoes a “hardening” process (increased resistance to proteolytic enzymes) in order to prevent subsequent sperm from penetrating. After fertilization, ZP goes through chemical and mechanical hardening; is thinner and has visco-elastic properties [1]. Mechanical ZP hardening may be evaluated by changes in ZP elasticity [2]. By using discrete continuum method [3-5], a model of discrete oscillatory spherical net of ZP is presented in ref [6]. Dynamics of oscillatory discrete spherical net ZP model is described by a system of differential equations along component displacement in radial directions w_{ij}, circular directions $u_{i.j}$ and meridian directions $v_{i.j}$ in the form:

$$m_{ij}\ddot{w}_{ij} = -c_{(w)ij}w_{ij} - \tilde{c}_{(w)ij}w_{ij}^{3} - b_{(w)ij}\ddot{w}_{ij} + F_{i,j}\cos\left(\Omega_{ij}t+\alpha_{ij}\right) + \tilde{F}_{i,j}\cos\left(\tilde{\Omega}_{ij}t+\tilde{\alpha}_{ij}\right) \qquad (1)$$

$$m_{ij}\ddot{u}_{ij} = +c_{(u)ij}\left(u_{i+1.j}-u_{i.j}\right) + \tilde{c}_{(u)ij}\left(u_{i+1.j}-u_{i.j}\right)^{3} - c_{(u)i-1,j}\left(u_{i.j}-u_{i-1.j}\right) - \tilde{c}_{(u)i-1,j}\left(u_{i.j}-u_{i-1.j}\right)^{3} - b_{(u)ij}\dot{u}_{ij} \qquad (2)$$

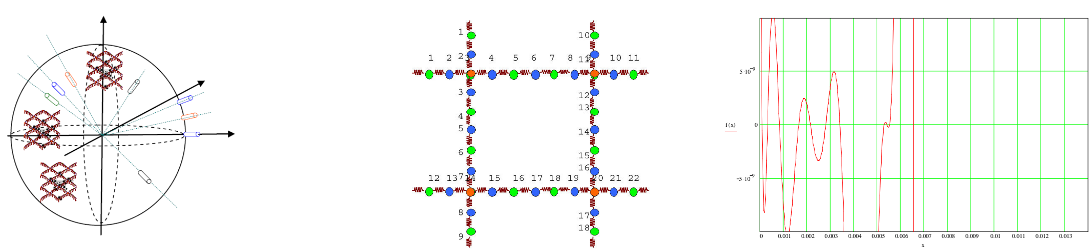
$$m_{ij}\ddot{v}_{ij} = +c_{(v)i,j}\left(v_{i.j+1}-v_{i.j}\right) + \tilde{c}_{(v)i,j}\left(v_{i.j+1}-v_{i.j}\right)^{3} - c_{(v)i,j-1}\left(v_{i.j}-v_{i.j-1}\right) - \tilde{c}_{(v)i,j-1}\left(v_{i.j}-v_{i.j-1}\right)^{3} - b_{(v)i,j}\dot{v}_{ij} \qquad (3)$$

For details see ref [6].

Frequency analysis of knot mass trajectory oscillations in oscillatory spherical net model of m ZP

Mass particle displacement in radial directions w_{ij}, circular directions $u_{i.j}$ and meridian directions $v_{i.j}$ are orthogonal. Previous system of differential equations is possible to solve as a three independent subsystems.

Subsystem (2) and (3) of differential equations are of the same type and presented mathematical description of mass particle motion in nonhomogenous chains. In Fig1a and b a part of spherical net ZP model is presented. For analysis of knot mass trajectory oscillations we consider a small part of this, the discrete spherical net presented in Fig 1b.



a* b* c*

Fig.1: **a*** Model of *ZP* spherical surface that shows radial direction of axis of constructive elements of the model - *ZP* proteins. **b*** Part of the *ZP* network on a part of the sphere (oocyte). Orange (*ZP*1), blue (*ZP*2) and green (*ZP*3) represents *ZP* proteins. Network is identical in circular and meridian direction. Axis shows directions of movements of *ZP* proteins. Each *ZP* protein is connected to the sphere with elastic springs that can oscillate in radial direction. ***c**** function from frequency equation presented in MathCAD

For one part of mechanical chain presented in FIg 1b that consists of 11 mass particles in circular and meridian direction we derived frequency equation in general, with arbitrary kinetic parameters. By using frequency equation and for specific distribution of mass particles [6] we determined a set of eigen circular frequencies of knot mass particles for free vibrations.

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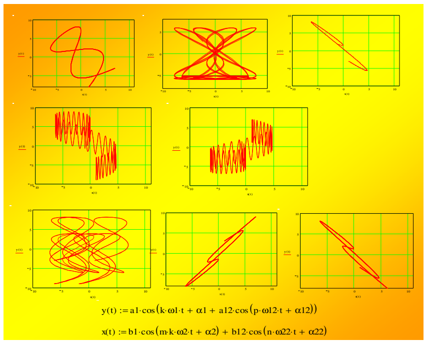
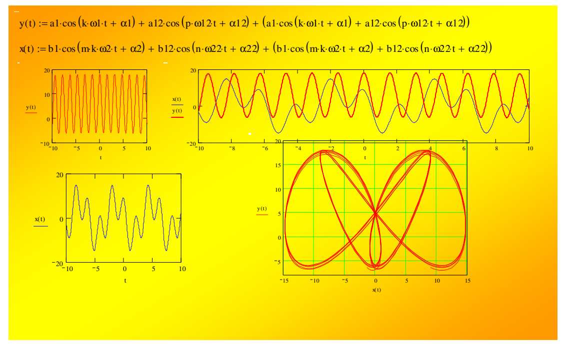
For each chain with eleven mass particles existes eleven eigen circular freguences. Each material particle oscillatory motion in direction of merdional /circular is in 11^{th} oscillatory regime in general case.
By using special initial conditions it is possible to filtrate the number of eigen circular frequencies and to reduce its number. In radial direction knot mass particle motion is one frequency in free regime of oscillations, and multyfrequency in forced regime. For finding eigen circular frequency of the mechanical chain with eleven degrees of freedom we use Mathcad. In figure 1c graphic of function from frequenty equation is presented in Matcad. Eleven roots of this function are square values of eleven eigen circulare frequencys of free vibrations.



a



b

Fig.2. 2a **.** A possible generalized Lissajous curves for illustration knot's mass particle trajectory motion - summarized displacements in two orthogonal directions: meridian and circular directions for different type of the two orthogonal mass particle component of oscillatory displacements: periodic attractors, chaotic, synchronized and synchronized cases. (Radial direction displacement is not included) b. A possible periodic attractor in the form of generalized Lissajous curve for illustration knot's mass particle trajectory periodic motion - summarized displacements in two orthogonal directions: meridian and circular directions. with component time history graphs for meridian and circular displacements of knot's mass particle (Radial direction displacement is not included).

CONCLUSIONS

From frequency analyses of knot mass particles for free and for forced vibrations we can conclude following: Component displacements in meridian and circular direction are multyfrequency. For the considered part of the net, knot mass particle oscillates in meridian and circular direction in multyfrequency regime with eleven frequencies. In this model knot mass particles oscillate in free oscillatory regime in radial direction with one eigen frequency; in forced regime knot mass particles oscillates in radial direction with number of external oscillation plus one.
Depending on initial perturbation of equilibrium state of material particles in spherical net, component displacement of knot mass particle are in multyfrequency oscillatory regime but with number of frequencies less or equal to the number of the frequencies of the system. Trajectory of resultant motion of knot mass particles depends on the excitated regime of oscillation by initial conditions and it could be in the form of periodical, non-periodical, stochastic like or chaotic –like trajectories.

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