WRINKLING MODE EVOLUTION IN HYPERELASTIC FILM/SUBSTRATE BILAYERS

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Summary Wrinkling modes are determined for a two-layer system comprised of an elastic film bonded to an infinitely deep pre-stretched elastomeric substrate with the entire bilayer undergoing compression. Nonlinear finite element simulations uncover advanced post-bifurcation modes. Both numerical simulations and theoretical analysis have been performed to reveal the underlying physics behind the wrinkling mode evolution.

INTRODUCTION

Surface wrinkling may occur when a composite system consisting of a hard skin on a soft underlayer is subject to in-plane compression. The early studies on this issue may date back to work in the middle of last century [1] aiming at the prevention of instability of engineering structures such as sandwich panels. Recent studies demonstrate that this surface wrinkling phenomenon of film/substrate systems finds such wide applications as in fabricating surfaces with controlled patterns with unique wetting, optical and acoustic properties, measuring the mechanical properties of materials and the design of flexible electronics [2-5]. Although theoretical studies during past years have led to procedures to determine the critical condition of instability and the corresponding wrinkling patterns, the post-buckling evolution of surface wrinkles is only recently being pursued. In this study, nonlinear finite element simulations uncover advanced post-bifurcation modes including period-doubling, folding and a newly identified mountain ridge mode. Numerical simulations and theoretical analysis enable us to understand the physics underlining the phenomena.

FINITE ELEMENT SIMULATIONS OF ADVANCED WRINKLING MODES

Plane strain finite element simulations of film-substrate systems with substrate pre-stretch have been carried out with the aim of revealing the ridge formation phenomenon and providing insights into when it should be expected to occur. Simulations have been performed using both the neo-Hookean and the Arruda-Boyce constitutive models for elastomers. The simulations are made with the commercial software, ABAQUS [6], and employing the hybrid element, CPE8MH, designed for simulations of incompressible materials. Displacement-controlled loading is employed with the horizontal displacement, u_1 (taken to be independent of the vertical coordinate x_2), and zero shear traction specified on the vertical sides of the model. Fig. 1, illustrates the evolution of the ridge mode as the system undergoes increasing compression beyond the onset of the classical sinusoidal wrinkling mode for a substrate that has been pre-stretched to $\lambda_1^S = 2$. The film to substrate ground state shear modulus ratio is $\mu_f / \mu_S = 836$ [7].

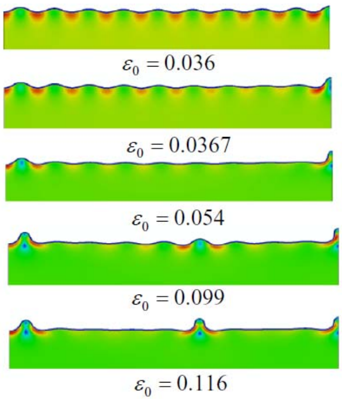

Fig. 1 Formation and evolution of localized ridges in a compressed neo-Hookean stiff film/compliant system under plane strain compression, ε_0 is the overall compressive strain.

When the substrate is pre-compressed to $\lambda_1^S = 0.7$ prior to attachment of the film and subsequent to imposition of the overall compressive strain ε_0, period-doubling followed by developing folds is observed (Fig. 2). Here the film and the substrate are modeled by the incompressible Arruda-Boyce model. The film/substrate modulus ratio is $\mu_f / \mu_s = 100$.

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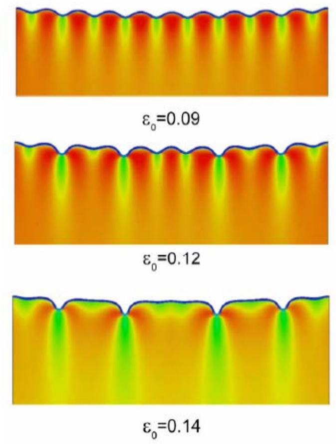

Fig. 2 A sequence of wrinkle patterns showing the emergence of period-doubling followed by developing folds.

EFFECT OF PRE-DEFORMATION ON NONLINEARITY OF SUBSTRATE STIFFNESS

To explore the effect of pre-deformation on the nonlinearity of the substrate nonlinearity, a computational is first developed. The calculation imposes a localized ridge-like or fold-like deformation on the surface of the uniformly deformed substrate and evaluates the energy required to create it. The displacements imposed on the surface are $u_1 = 0$ and $u_2 = 4\xi L e^{-(X/L)^2}$, with $\xi > 0$ corresponding to a ridge and $\xi < 0$ to a fold. Denote the work done to impose the displacements by $\Delta\Xi$. Equivalently, this is the change in elastic energy in the substrate. When the half-space has been subject to a pre-stretch, $\lambda_0^S = 2$, the ratio of the energy to create a fold-like deformation to that to create a ridge-like deformation (both with amplitude $|\xi| = 0.4$) is

$$\frac{(\Delta\Xi)_{\xi=-0.4}}{(\Delta\Xi)_{\xi=+0.4}} = 1.13 \tag{1}$$

When the substrate is subject to a pre-compression, $\lambda_0^S = 0.6$, the same ratio is

$$\frac{(\Delta\Xi)_{\xi=-0.4}}{(\Delta\Xi)_{\xi=+0.4}} = 0.92 \tag{2}$$

These results suggest the following: a pre-stretch favors the occurrence of outward deflections of the surface (e.g., ridge formation) while pre-compression favors inward deflections (e.g., folds). To get further physical insight into the nonlinear wrinkling behavior of a stiff film on a compliant substrate, a differential equation model of the film/substrate system is proposed, which enhances earlier models used to explore advanced modes by including the tangential restraint of the substrate on the film. The model reveals the important role played by this restraint on the advance modes. In particular, it shows that the localization aspect of the advanced modes is suppressed if the tangential stiffness exceeds a critical level [8]. One-dimensional models such as this are mainly limited by the difficulty of accurately representing the nonlinearity of a substrate material, which is generally quite complex. Nevertheless, the models can provide important qualitative insights to the highly nonlinear wrinkling phenomena.

CONCLUSIONS

The elastomeric film/substrate bilayer admits a rich variety of wrinkling modes, especially when pre-stretch of the substrate is considered. Our study reveals that the pre-deformation in the substrate leads to nonlinearity in the substrate stiffness which regulates the wrinkling mode evolution.

References

[1] Allen H. G.: Analysis and Design of Structural Sandwich Panels, Pergamon, New York, 1969.
[2] Bowden N., et al.: Spontaneous formation of ordered structures in thin films of metals supported on an elastomeric polymer. *Nature* 393: 146-149, 1998.
[3] Stafford C. M. *et al.*: A buckling-based metrology for measuring the elastic moduli of polymeric thin films. *Na. Mater.* **3**: 545-550, 2004.
[4] Lacour, S. P. et al.: Stretchable interconnects for elastic electronic surfaces. *Proc. IEEE* 93: 1459-1467, 2005.
[5] Khang D. Y.: A stretchable form of single-crystal silicon for high-performance electronics on rubber substrates. *Science* 311: 208-212, 2006.
[6] ABAQUS: ABAQUS analysis user's manual, version 6.8, 2008.
[7] Cao Y. P., Hutchinson J. W.: Wrinkling phenomena in neo-Hookean film/substrate bilayers. J. Appl. Mech. (in press)
[8] Zeng J. F., Zhao X. H., Cao Y. P., Hutchinson J. W.: Localized ridge wrinkling of stiff films on compliant substrates. (submitted)