MODELING THE INFLUENCE OF MECHANICAL FACTORS ON LONGITUDINAL BONE GROWTH

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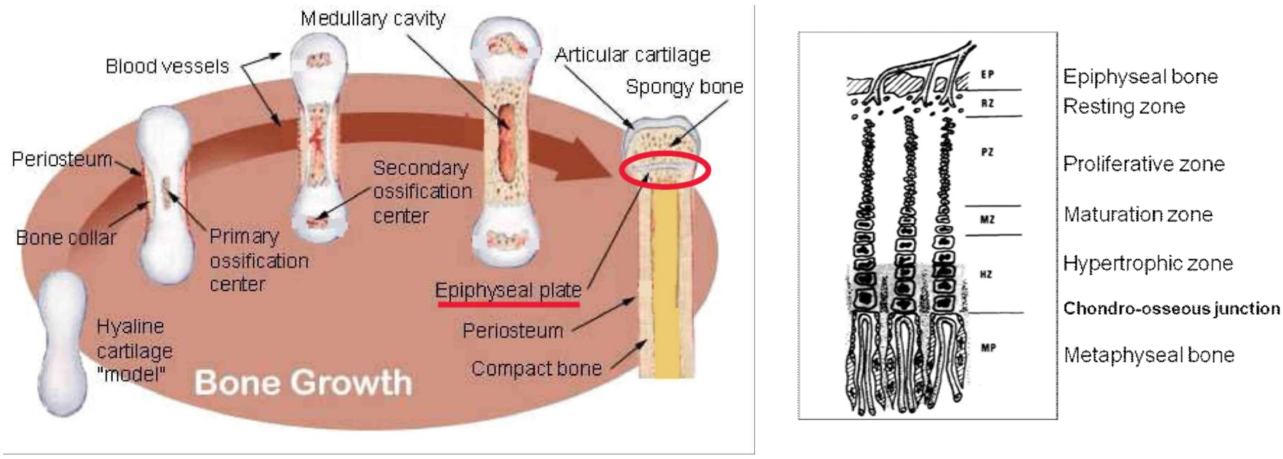
Summary A model is developed to investigate the influence of mechanical factors and a biological growth parameter on the stationary movement of the ossification interface in the growth plate, its orientation and stability. Critical loading parameters, the parameter of biological growth and the rate of movement of the ossification interface at which the change of the orientation and the loss of stability of the ossification interface occur are obtained.

INTRODUCTION

Longitudinal bone growth in skeletons of vertebrates (Fig. 1(a)) is carried out mainly by the processes of growth and ossification in the growth (epiphyseal) plate. The growth plate is a layer of hyaline cartilage between metaphysis and epiphysis of a long bone (Fig. 1(b)). Long bones grow in length due to the following processes: (a) the increase in the thickness of the growth plate due to proliferation (reproduction of new cells) and hypertrophy (increase in size) of cartilage cells, and (b) ossification process: when growing cells from the hypertrophic zone secrete substances to the surrounding matrix that calcifies it, the calcified matrix ceases the supply of nutrients to the growing cartilage cells and the cells start to starve and as a result they die releasing the place for bone cells that are arriving with growing from the down of the growth plate blood vessels. The surface between the growing cartilage cells in the matrix (we will call this tissue as "cartilage") and the dead cartilage cells in the calcified matrix ("bone") is called an ossification interface (a chondro-osseous junction). It is well known nowadays that the growth and the ossification processes are directed, besides genetic, hormonal, metabolic, vascular, also by mechanical factors [1].



(a) Endochondral ossification in long bones [2]

(b) Diagram of a growth plate [3]

Figure 1. Bone growth and a growth plate

Development of models for the description of the influence of mechanical loadings on longitudinal bone growth can help in children chirurgical orthopaedy where for the cure of the growth plate abnormalities in particular mechanical loadings are applied. Accordingly, it is important to know a magnitude and a type of loadings at which, for example, the premature closure of the growth plate does not occur. Closure of the growth plate is a completion of a bridging mechanism by which bony bridges grow longitudinally across the growth plate. Excessive loading can cause rapid cessation of physeal growth, fissuration of the ossification interface leading to the bridges formation and the premature closure of the growth plate [4].

TWO-PHASE STATES AND KINETIC STABILITY OF STATIONARY MOVEMENT OF THE OSSIFICATION INTERFACE

The growth plate is modeled by a layer of "cartilage" of the thickness h located in an infinite medium that is "bone" (Fig. 2). We consider the period of time when the thickness h remains constant, that is two competitive processes

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take place: the biological growth in the growth plate (due to proliferation and hypertrophy of cartilage cells) and the ossification process. We model the former as a volumetric growth of cartilage tissue, the latter as a stationary movement of the ossification interface.

The rate of the stationary movement of the ossification interface V_m^Γ according to the linear approximation of the second law of thermodynamics is defined by the jump in chemical potentials μ_{bone} and μ_{cart} of "bone" and "cartilage"

$$V_m^\Gamma = -k(\mu_{bone} - \mu_{cart}), \quad k = const > 0, \quad V_m^\Gamma = const \tag{1}$$

Assuming the displacement continuity across the ossification interface, we define the difference of the chemical potentials as

$$\mu_{bone} - \mu_{cart} = \mathbf{m} \cdot [\![\mathbf{M}]\!] \cdot \mathbf{m}, \quad \text{where } \mathbf{M} = W\,\mathbf{I} - \mathbf{F}^T \boldsymbol{S}, \tag{2}$$

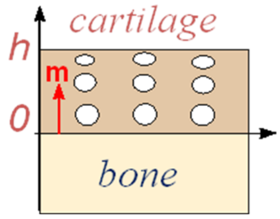

Figure 2. Model of the growth plate.

$\mathbf{M}, \mathbf{F}, \mathbf{m}, \mathrm{W}(\mathbf{F}), \mathbf{S} = \partial W / \partial \mathbf{F}$ are the Eshelby stress tensor, the deformation gradient, the normal to the ossification interface in the reference configuration, the strain-energy function and the first Piola-Kirchhoff stress tensor, respectively. The brackets $[\![\cdot]\!] = (\cdot)_+ - (\cdot)_-$ denote a jump across the ossification interface, subscripts "+" and "–" correspond to "bone" and "cartilage", respectively. So we have the following system of jump conditions across the ossification interface

$$[\![\mathbf{u}]\!] = 0, \quad [\![\mathbf{S}]\!]\mathbf{m} = 0, \quad [\![W]\!] - \mathbf{S}_\pm : [\![\mathbf{F}]\!] + \frac{V_m^\Gamma}{k} = 0 \tag{3}$$

The first two equations are the displacement and the traction continuity conditions, respectively; the third one is a result of substitution of (2) into (1).

We determine two-phase stationary states satisfying system (3) and the conditions of the loss of stability of the stationary movement of the ossification interface in dependence on a strain state, the volumetric growth and V_m^Γ. For the investigation of the stability we use a generalization of the kinetic criterion developed in [5], [6] that implies instability of two-phase states if superimposed perturbations of the stationary moving ossification interface and the displacements grow. Also for various values of the volumetric growth and V_m^Γ we construct transformation surfaces determining possible strains on the ossification interfaces and their orientations. The technique for the construction of the transformation surfaces was developed earlier for the case of the equilibrium interfaces (e.g. [7], [8]), in the present paper we considered a generalization for the case of the stationary moving ossification interfaces taking into account the volumetric growth.

CONCLUSIONS

For a number of constitutive relations for "cartilage" and "bone" it is shown the following:

- Analysis of the solution of the equations on the ossification interface demonstrates that stretching leads to the increase of V_m^Γ, compression - to decrease of V_m^Γ that is in agreement with experimental data (e.g., [1])
- Under compression the ossification interface orientates under an angle to the vertical axis of bone, under stretching – perpendicular to this axis.
- Loadings bigger critical values lead to the loss of stability of the ossification interface that as we suppose can induce further the "bridges formation" and the premature closure of the growth plate. The loss of stability occurs earlier for smaller V_m^Γ. Increasing of the volumetric growth can exert both the stabilizing and destabilizing effect depending on a chosen strain-energy function.

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