Elastic and geometrical effects in the evolution of living materials: A configurational analysis of growth and mass transport

Pasquale Ciarletta[*] [a)], Luigi Preziosi[**] & Gerard A. Maugin [*]

[*] *Université Pierre et Marie Curie-Paris 6, Institut Jean Le Rond d'Alembert (UMR CNRS 7190), Case 162, 4 place Jussieu, 75252 Paris cedex 05, France*
[**] *Dipartimento di Matematica, Politecnico di Torino, c.so Duca degli Abruzzi 24, Torino 10123, Italy*

Summary In this work, we define a thermo-mechanical framework for understanding the interrelations and the coordination mechanisms between the geometry and the mechanical/biochemical state of a growing biological material. Bridging the gap from micro-structural changes to macroscopic rearrangements, a continuous configurational theory is defined, aimed at understanding how elasticity, geometry and biochemical feedbacks coordinate growth and mass transport phenomena.

In the biological realm, mass exchanges and diffusion of species are physiological necessities due to non-equilibrium transport and reaction equations of biochemicals at the microscale. Taking cancer development as an illustrative example, nutrients/growth factors diffuse through the interstitial liquid and are uptaken by tumour cells for duplicating. This uncontrolled proliferation is often initiated by the deregulation of cell adhesion proteins binding to other host cells, while further invasion can be promoted by disruption and plastic reorganization of the extra-cellular matrix in the surrounding host tissue [1]. Even if continuous growth theories have been proposed to incorporate the diffusion of biochemical species [2], multiphase approaches received a greater interest in mathematical modeling of biological systems, providing a straightforward description of the biochemical growth processes and the mass exchanges between the fluid and solid components of the material [3]. Nevertheless, major drawbacks of mixture models consist in the definition of constitutive equation for the partial stresses and of boundary conditions for the phases.

In the last decades, a considerable amount of modeling research in continuum biomechanics has been performed to accommodate the evolution laws of mass variations inside a biological material, written as a volumetric change of bulk material or an accretion/resorption at a surface. Embodying the concept of multiplicative decomposition of finite strains, Epstein and Maugin [4] first emphasized the role played by the elastic stress in directing a thermodynamically admissible evolution of volumetric growth. Introducing a surface strain energy density, the growth of elastic surfaces can be similarly related to the action of configurational forces of the Eshelby type [5,6]. These continuous approaches exclude *ab initio* the consideration of mass diffusion phenomena, which are out of reach for a first-order material isomorphism. Despite a great geometrical complexity in the accompanying framework, Ciarletta and Maugin [7] have recently proposed the bases for a strain-gradient theory for material growth and remodeling, and the material Eshelby hyperstress is found to drive the evolution of curvature-related inhomogeneities in a second order transplant. The driving forces of mass diffusion can be written in terms of covariant material derivatives reflecting, in a purely geometrical manner, the presence of a (first-order) torsion and a (second-order) curvature [8]. Therefore, it is now well accepted that combinations of mechanical, geometrical and biochemical feedbacks build the unknown regulation mechanisms in morphogenetic processes.

In this work, we define a thermo-mechanical framework for understanding the interrelations and the coordination mechanisms between the mechanical and the biochemical states of a growing biological material. Bridging the gap from micro-structural changes to macroscopic rearrangements, a continuous configurational theory is defined, aimed at understanding the mechanical and biochemical feedbacks coordinating the evolution of growth and mass transport processes. Whilst volumetric growth is described at the expense of the material porosity and/or as a smooth change in a single-phase material, surface growth is modeled as a phase transition phenomenon forcing the evolution of a non-material interfacial structure. In

a) Corresponding author. Email: pasquale.ciarletta@upmc.fr

practice, surface growth is regarded as the ensemble of the phase transformation phenomena driving the permeability and the absorbing properties in a very narrow volume across a material interface. Disregarding the microscopic variability, the physical properties inside this growing layer are modeled by associating corresponding surface fields. In modeling terms, we introduce a non-material interface carrying thermo-mechanical properties, which can be treated as a moving discontinuity inside the continuous biological system. Extending the original idea of Gibbs [9], balance principles for the surface and the volume fields are formulated for imposing the conservation of the thermo-mechanical properties of the entire biological system. The balance laws of the pseudomementum densities, in particular, identify the Eshelbian stress measures and the driving forces determining the evolution of the material inhomogeneities.

The proposed theoretical framework is applied in few system models, discussing the extent of its results for analyzing the emergence and the regulation of shapes in living organisms. The biomechanical modeling of both geometry- and stress-dependent effects on growth not only allows better understanding of morphogenetic processes, but has also important biomedical applications, e.g. for optimizing scaffolds in regenerative medicine and tissue engineering.

Acknowledgement.
A European Community funding (Marie Curie grant ERG-256605) is gratefully acknowledged.

References

[1] Preziosi L., ed., Cancer Modelling and Simulation, Chapman Hall/CRC Press, 2003.

[2] Ambrosi, D., Guana, F.: Stress modulated growth. Math. Mech. Solids **12**, 319-343, 2007.

[3] Chauvière, A., Preziosi, L., Verdier, C., eds., Cell Mechanics: From Single Scale-Based Models to Multiscale Modeling, CRC Press, 2010

[4] Epstein, M., Maugin, G.A.: Thermomechanics of volumetric growth in uniform bodies, Int. J. Plasticity **16**, 951–978, 2000.

[5] Ganghoffer, J.F.: Mechanical modeling of growth considering domain variation Part II: volumetric and surface growth involving Eshelby tensors. J. Mech. Phys. Solids **58**, 1434-1459, 2010.

[6] Maugin G.A.: Configurational Forces: Thermomechanics, Physics, Mathematics and Numerics, CRC/Taylor and Francis, Boca Raton, Florida, 2011.

[7] Ciarletta, P., Maugin, G.A.: Elements of a finite strain-gradient thermomechanical theory for material growth and remodeling. Int. J. NonLinear Mech. **46**, 1341–1346, 2011.

[8] Ciarletta, P., Ambrosi, D., Maugin, G.A.: Mass transport in morphogenetic processes: A second gradient theory for volumetric growth and material remodeling. J. Mech. Phys. Solids 2012, in press (doi:10.1016/j.jmps.2011.11.011)

[9] Gibbs, J.: Collected works, Longmans Green and Co., vol I-II, New York, 1928.