GROWTH AND MECHANICAL RESPONSES OF THE ARTERIAL WALL

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Summary A two-layer thick-walled circular cylindrical tube biomechanical model is proposed and growth, residual stresses and the mechanical responses of the arterial wall are examined. The fields of the displacements and the distribution of stresses for the arterial wall with growth are solved analytically. Effects of growth on the mechanical responses and residual stresses of the arterial wall are discussed.

INTRODUCTION

It has been generally agreed that mechanics plays an essential role in the process involved in vascular development and disease, and growth can play a key role in the stability of a biological structure [1,2]. There is a strong coupling between mechanics and growth which is linked by residual stresses [3]. So it is vital to understand well the relationship in arteries between the distribution of residual stresses due to growth and the mechanical properties of the arterial wall [4].

BASIC RELATIONS

The arterial wall may be modeled as a two-family fiber-reinforced incompressible composite material tube [3]. The strain energy function for the media considering the activation of the smooth muscle is

$$W=\sum_i \frac{\mu_i}{\alpha_i}\left(\tilde{\lambda}_r^{\alpha_i}+\tilde{\lambda}_\theta^{\alpha_i}+\tilde{\lambda}_z^{\alpha_i}-3\right)+\frac{c_1}{c_2}\left\{\exp\left[c_2\left(I_4^*-1\right)^2\right]-1\right\}+T_m\left\{\tilde{\lambda}_\theta+\frac{1}{3}\frac{\left(\tilde{\lambda}_m-\tilde{\lambda}_\theta\right)^3}{\left(\tilde{\lambda}_m-\tilde{\lambda}_0\right)^2}\right\} \tag{1}$$

in which, $\alpha_i, \mu_i, c_1, c_2, T_m, \lambda_m, \lambda_0$ are material constants. $\tilde{\lambda}_r, \tilde{\lambda}_\theta, \tilde{\lambda}_z$ are principal stretches of the elastic deformation tensor. $I_4^*=kI_1+(1-3k)I_4$, $I_1=\tilde{\lambda}_r^2+\tilde{\lambda}_\theta^2+\tilde{\lambda}_z^2$, $I_4=\sin^2\alpha\tilde{\lambda}_\theta^2+\cos^2\alpha\tilde{\lambda}_z^2$. $\mathbf{M}_\alpha=[0,\sin\alpha,\cos\alpha]$ are fiber orientations in the reference configuration and α is the angle between the direction of the fibers and the axial direction of the arterial wall. k is the dispersion parameter. The strain energy function for the adventitia is without the third item of (1).

The deformation function of the tube subjected to the inner inflation and the axial stretch is

$$r=r(R)>0,\ \theta=\frac{\pi}{\Theta_0}\Theta,\ z=\lambda_z Z \tag{2}$$

$r(R)$ is a function to be determined, $\pi-\Theta_0$ denotes the opening angle. The deformation-gradient tensor $\mathbf{F}=\mathrm{diag}(\lambda_r,\lambda_\theta,\lambda_z)$ can be decomposed into $\mathbf{F}=\mathbf{A}\cdot\mathbf{G}$. $\mathbf{G}=\mathrm{diag}(g_1,g_2,g_3)$ is the growth tensor, $\mathbf{A}=\mathrm{diag}(\tilde{\lambda}_r,\tilde{\lambda}_\theta,\tilde{\lambda}_z)$ is the elastic deformation tensor. The equilibrium equations for the tube in the absence of body forces are

$$\frac{\mathrm{d}\sigma_{rr}}{\mathrm{d}R}+\frac{r'(R)}{r(R)}\left[\sigma_{rr}-\sigma_{\theta\theta}\right]=0 \tag{3}$$

The boundary conditions are

$$\sigma_{rr}^m=-q_0, R=R_i; \sigma_{rr}^a=0, R=R_0 \tag{4}$$

where, q_0 is the inner inflation pressure. The continuity of the radial stress σ_{rr} at the interface implies

$$\sigma_{rr}^m=\sigma_{rr}^a, R=R_m \tag{5}$$

SOLUTION AND RESULTS

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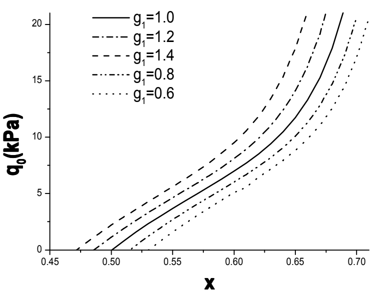

Fig.1. Pressure-radius curves with growth

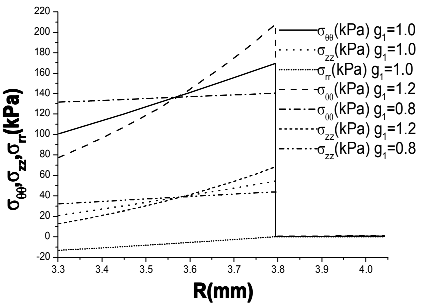

Fig.2. Stress distribution with growth

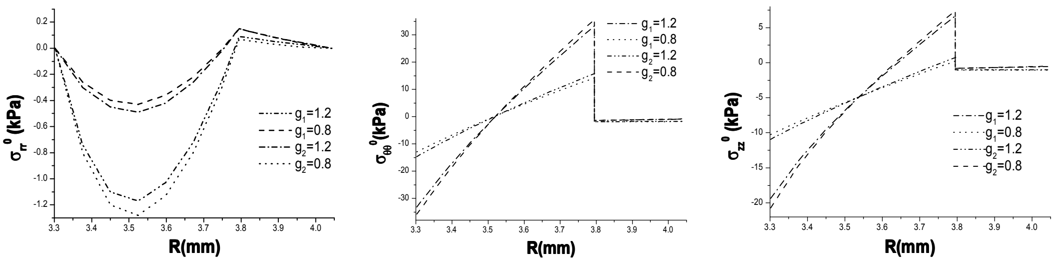

Fig.3. Distributions of residual stresses

The mechanical effect of growth is evident and growth can have both stiffening and weakening effects. The inner surface of the arterial wall will have a larger stretch with a radial shrinking or a circumferential growth . But it will have a smaller stretch with radial growth or a circumferential shrinking.

The effect of growth on the distribution of the circumferential stress and the axial stress of the arterial wall is obvious. With a radial growth or a circumferential shrinking, the distribution of stresses will become more inhomogeneous. However, with a radial shrinking or a circumferential growth, the stress distribution will become more homogeneous with the increasing growth factor until the factor attains a certain value.

Both the residual circumferential stress and the residual axial stress increase with increase in radius in the media. They are compressive stress at the field near the inner surface while they become tensile stress near the interface of the two layers. The maximal values for the compressive residual circumferential stress and the compressive residual axial stress occur at the inner surface. At the same time, the maximal values for the tensile residual circumferential stress and the tensile residual axial stress occur at the interface of the two layers. They are compressive stresses in the adventitia and their absolute values decrease with increase in radius in the adventitia.

CONCLUTIONS

The mechanical response of the arterial wall with growth and residual stresses due to growth may be described by the model presented in this paper. The deformation and distributions of stresses for the arterial wall, the residual stresses due to growth may be obtained. The mechanical effect of growth on the deformation and the distributions of the circumferential stress and the axial stress of the arterial wall is obvious.

References

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