AN ANISOTROPIC DISCRETE FIBER MODEL BASED ON A GENERALIZED STRAIN INVARIANT WITH APPLICATION TO SOFT BIOLOGICAL TISSUES

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Summary A discrete fiber model has been developed to simulate the anisotropic nonlinear anisotropic response observed in fibrous connective tissues.

INTRODUCTION

Lanir [1] introduced a continuum model of fibrous connective tissues, which proposed that the tissue is a composite of fiber bundles and an isotropic matrix. In its simplest form the matrix is modeled as an incompressible fluid and the fibers are modeled using a distribution function that characterizes the orientation of an infinite number of identical fiber bundles. In addition, each fiber bundle is characterized by an undulation distribution that determines when individual fibers in the fiber bundle first become stretched and begin to resist additional stretching.
Flynn et al. [2] have simplified the model in two ways: (a) the influence of the fiber orientation distribution is replaced by weighted responses of six discrete fiber bundles that are oriented in equally spaced directions defined by a regular icosahedron [3]; and (b) the undulation distribution is replaced by a simple step distribution that leads to a closed form analytical expression for the strain energy function of the tissue. Although the model in [2] was successfully used to model anisotropic response in a number of experiments on different tissues, it was observed that when the weights of the discrete fiber bundles are equal then the model does not reduce to a an isotropic model for general undulation distributions. This undesirable anisotropy was further studied in [4].

PROPOSED MODEL

The objective of this research is to develop a new discrete fiber model for nonlinear anisotropic elastic response of tissues which can reduce to an isotropic model when the weights are equal even for general nonlinear strain energy functions. Specifically, motivated by the work in [3] and [5] a generalized invariant γ is proposed by the expressions

$$\gamma = \sum_{i=1}^{6} w_i \, [(\mathbf{C}+\mathbf{C}^{-1}) \bullet \mathbf{B}_i - 2] \, , \quad \sum_{i=1}^{6} w_i = 1 \, , \; w_i \geq 0 \, , \quad \sum_{i=1}^{6} \mathbf{B}_i = 2\,\mathbf{I} \, , \tag{1}$$

where $\mathbf{C}$ is the right Cauchy-Green deformation tensor, w_i are weights and $\mathbf{B}_i$ are symmetric structural tensors based on directions of a regular icosahedron, and $\mathbf{A} \bullet \mathbf{B} = \mathrm{tr}(\mathbf{A}\mathbf{B}^T)$ denotes the inner product between two second order tensors {$\mathbf{A}$ $\mathbf{B}$}. Also, when the matrix material is modeled as an incompressible inviscid fluid the strain energy function Σ (per unit mass) for the discrete fiber model is proposed as an M'th order polynomial

$$\rho_0 \Sigma = \frac{1}{2} K \, [\gamma + \sum_{m=2}^{m} \frac{\gamma_m}{m} (\frac{\gamma}{\gamma_m})^m] \, , \tag{2}$$

where ρ_0 is the mass density of the material in its stress-free reference configuration, K is a material constant having the units of stress, γ_m are dimensionless material constants which determine the value of γ when the m'th term becomes significant.

When the weights w_i are equal γ becomes an isotropic invariant which describes isotropic response for any strain energy function $\Sigma(\gamma)$. Consequently, inequality of the values of w_i is a pure measure of anisotropy in the material.

RESULTS

Examples show that the model can accurately simulate the nonlinear anisotropic response observed in: uniaxial tensile loading of pig skin; biaxial stretching of rabbit skin (Fig. 1); simple shear of septal myocardium; and equibiaxial loading of fresh and fixed aortic valve cusps. The predicted fiber orientations are in general agreement with measurements reported in the literature.

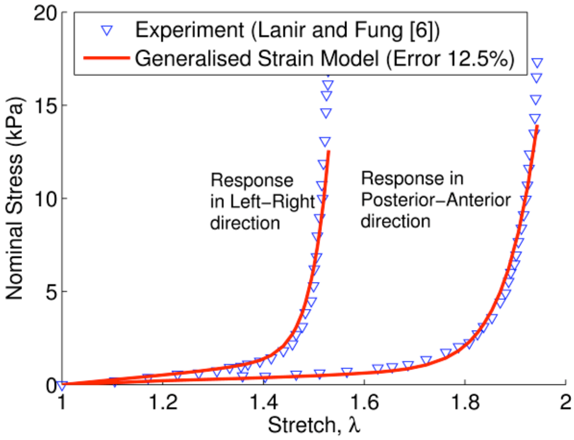

Fig. 1: Simulation of biaxial stretching of rabbit skin.

CONCLUSIONS

A discrete fiber model has been developed to simulate the nonlinear anisotropic response observed in fibrous connective tissues. The strain energy is a nonlinear function of a generalized invariant, which models weighted effects of six fiber bundles oriented in directions based on a regular icosahedron. The proposed model clearly separates the effects of anisotropy, which is characterized by unequal weights, from the nonlinear elastic response of individual fiber bundles. Examples show that the model successfully simulates observed response in experiments.

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