AN EXPERIMENTAL AND THEORETICAL STUDY OF SIZE EFFECTS IN THE TORSION OF MICRO-SIZED METALLIC WIRES

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Abstract Both torsion and tensile tests are performed on polycrystalline copper wires with diameters in the range of 20-50μm. Significant size effects are observed at initial yield and plastic flow stress in torsion, while only a minor size effect presents in tension. The experimental observations are explained in terms of the Fleck-Willis strain gradient plasticity theory. These results confirm that a considerable strain-gradient still exists in the torsion of micro-sized polycrystalline wires, overriding the influence of external geometrical size and grain size.

INTRODUCTION

In the last twenty years many experimental observations presented pronounced size effects when the characteristic length scale associated with non-uniform is in the order of microns. Torsion of thin metallic wires is recognized as an excellent approach to explore the mechanical behavior of small-scaled materials. It covers the entire deformation range, from elastic deformation, through yielding, to the strain-hardening regime, and for a combination of structural and grain sizes, whereby the strain gradients are naturally generated in plasticity. Fleck *et al*. [1] performed torsion tests on polycrystalline copper wires with diameters ranging from 12 to 170 μm and found the normalized torque in the plastic range increases dramatically with decreasing wire diameters, while no significant size effect on the tensile properties was observed. However, their data do not have enough resolution to display the elastic limit. More recent torsion experiments on micro-sized copper wires performed by Dunstan *et al*. [2] and Liu *et al*. [3] show comparable size effects at both incipient yield and plasticity. But the technique Dunstan *et al*. used can not directly measure the torque acting on specimen. Moreover, explanations for such effects are controversial, including strain gradient [1, 4-6], stress gradient [7] and the critical thickness effect [2]. By comparison, phenomenological strain gradient theories of isotropic plasticity represent a relatively simple approach to predict such size effects, in which the spatial gradients of plastic strain associated with the geometrically necessary dislocations lead to enhanced hardening.

EXPERIMENTAL AND RESULTS

The commercially cold-drawn polycrystalline copper wires (purity 99.99%) with diameters ranging from 20 to 50 μm were annealed at 410℃ for 2.4 h in a vacuum furnace to remove internal stress from the fabrication and to ensure they possess similar grain sizes. The tensile and torsional responses were then measured at a strain rate below 10^{-3}/s. Details of the experimental methods are given in literature [3]. Representative uniaxial tension data for the copper wires are shown in Fig. 1 and the corresponding torsion responses are given in Fig. 2. Generally, we performed at least three tests of each type for each diameter of wire. For the torsion data, we used the mean curve of the three test curves as a representative of the torsion response of each diameter copper wire. We conclude from Fig. 1 that the uniaxial stress is elevated slightly with decreasing wire diameter, which indicates that the influence of sample dimension, microstructure size or surface layers on tensile behavior is small at this scale. No systematic trend between tensile stress-strain curve and diameter appears in the experiment.

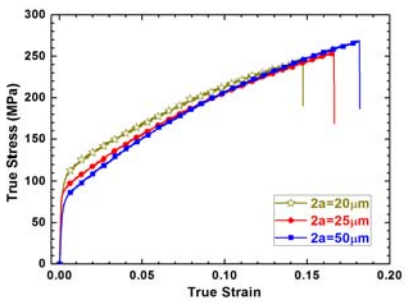

Fig.1 Tensile true stress-strain curves for copper wires.

Following Fleck *et al*. [1], the torsion data in Fig. 2 have been displayed in the form of normalized torque Q/a^3 versus surface shear strain κa. Here, Q is the torque, κ denotes the twist per unit length and a the wire radius. For macroscopic plasticity, dimensional analysis implies Q/a^3 versus κa is independent of wire radius. But here are two particularly interested features: (i) a pronounced elevation of the initial yielding is present, and (ii) a strong size effect emerges in the plastic flow stress. For example, at κa=0.05, the value of Q/a^3 for $2a$=20μm is appropriately 1.6 times of that for $2a$=50μm. Unlike the torsion data in previous experiments [1, 2], the wire-torsion tests here are straightforward and have high resolution in the low-strain region. It is acknowledged that the wavelets in the curves are caused by the slight misalignment between the specimen and the toque meter during testing.

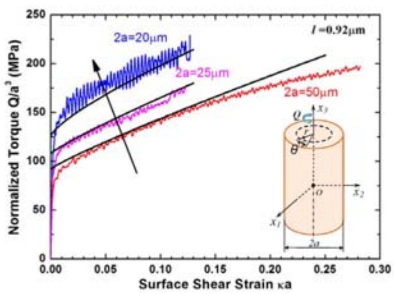

Fig.2 Torsional responses of copper wires with diameters in the range 20-50μm.

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THEORETICAL ANALYSIS

In this section, the new torsion data are analysed by means of a strain gradient plasticity theory proposed by Fleck and Willis [4, 5]. The incremental higher-order theory assumes associated plastic flow and possesses a convex yield surface. To simplify the analysis and obtain the closed-form expressions, the wires are assumed to be isotropic, incompressible and rigid-plastic. In this case, the plastic behavior can be characterized by the plastic strain rate $\dot{\varepsilon}_{ij}^{\mathrm{pl}}$ and its spatial gradient $\dot{\varepsilon}_{ij,k}^{\mathrm{pl}}$. And the material length scales are involved naturally. We restrict attention to the simplest model that contains one single material length scale *l*. Following Fleck and Hutchinson [6], the generalized overall effective plastic strain rate $\dot{E}_P$ takes the form

$$\dot{E}_{\mathrm{P}} = \left[\dot{\varepsilon}_{\mathrm{P}}^{\mu} + (l\dot{\varepsilon}_{\mathrm{P}}^{*})^{\mu} \right]^{\frac{1}{\mu}} \tag{1}$$

Here, $\dot{\varepsilon}_{\mathrm{P}} = \sqrt{\frac{2}{3}\dot{\varepsilon}_{ij}^{\mathrm{pl}}\dot{\varepsilon}_{ij}^{\mathrm{pl}}}$ is the standard Von Mises plastic strain rate used to consider the plastic dissipation results from the motion of statistically stored dislocations (SSDs), while $\dot{\varepsilon}_{\mathrm{P}}^{*} = \sqrt{\frac{2}{3}\dot{\varepsilon}_{ij,k}^{\mathrm{pl}}\dot{\varepsilon}_{ij,k}^{\mathrm{pl}}}$ is the rate of plastic strain gradient used to represent the contribution of geometrically necessary dislocations (GNDs). And the exponent μ is an additional material parameter dependent on these two contributions. A Cartesian reference frame (x_1, x_2, x_3) and a cylindrical polar coordinate system (r, θ, x_3) shown in the inset of Fig.2 are introduced. The generalized overall effective plastic strain rate is then given by

$$\dot{E}_{\mathrm{P}} = \frac{\dot{\kappa}}{\sqrt{3}}\left[r^{\mu} + \left(\sqrt{2}l\right)^{\mu} \right]^{\frac{1}{\mu}} \tag{2}$$

where $\dot{\kappa}$ is the twist rate per unit length. The plastic response considering of strain gradients for the rigid-hardening solid can be identified as

$$\Sigma_f = \sigma_{\mathrm{ref}} + \sigma_0 E_P^N \tag{3}$$

where $E_P = \int \dot{E}_P \mathrm{d}t$ is the accumulated effective plastic strain. If the strain gradient effect is absent, Eq. (3) is degenerated to a conventional power-law relation. And σ_{ref} is the initial yield stress, σ_0 is a measure of the hardening modulus and N is the hardening exponent. These parameters can be obtained by fitting the tensile data, as listed in Table 1. In the wire torsion, the torque Q is the work-conjugate to κ, so that the external work rate is $Q\dot{\kappa}$. And the torque is given by

$$Q = \frac{1}{\dot{\kappa}}\int_V \Sigma_f \dot{E}_P \mathrm{d}V = \frac{2\pi}{\dot{\kappa}}\int_0^a \Sigma_f \dot{E}_P r \,\mathrm{d}r \tag{4}$$

Finally, substituting Eqs. (2) and (3) into Eq. (4) with the assumption of $\mu = 2$, one can obtain the normalized torque

$$\frac{Q}{a^3} = \frac{6\pi\sigma_0(\kappa a)^N}{N+3}\left\{ \left[\tfrac{1}{3} + \tfrac{2}{3}(l/a)^2\right]^{\frac{N+3}{2}} - \left[\sqrt{\tfrac{2}{3}}(l/a)\right]^{N+3} \right\} + \frac{2\sqrt{3}\pi\sigma_{\mathrm{ref}}}{9}\left\{ \left[1 + 2(l/a)^2\right]^{\frac{3}{2}} - 2\sqrt{2}(l/a)^3 \right\} \tag{5}$$

If the torsional response curve of 2a=20μm was used as a calibration curve, we got *l*=0.92μm. The comparisons of theoretical predicts with the experimental results are shown in Fig. 2.

Table 1. Specification of tensile stress-strain curves

$2a$ (μm)	σ_{ref} (MPa)	σ_0 (MPa)	N
20	102	652.4	0.77
25	88	737.5	0.88
50	76	695.2	0.89

CONCLUSIONS

We observed significant size effects at initial yield and plastic flow stress in the torsion of micro-sized polycrystalline copper wires, while only a minor size effect in tension. The experimental results agree well with the theoretical predilections based on the strain gradient plasticity theory proposed by Fleck and Willis.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (No.11072084).

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