VIBRATION ISOLATION AND CONTROL OF AN IMPACT OSCILLATOR

James Ing[a)]*, Elena Sitnikova*, Ekaterina Pavlovskaia*, Yang Liu* & Marian Wiercigroch*
*Centre for Applied Dynamics Research, School of Engineering, University of Aberdeen, Aberdeen, AB24 3UE, UK

Summary Impact oscillators are known to exhibit strongly nonlinear behaviour, such as period adding, nonlinear resonances and abrupt routes to chaos. Typically these result from an interaction between nonsmooth events, i.e. grazing of the impact surface, and the inherent smooth nonlinearity due to the intermittent contact. Some experimental results of an impact oscillator are introduced, demonstrating co-existence of attractors and chaotic response. Two different forms of control are then used to obtain a favorable response; An impulsive control which exploits co-existence of stable solutions to minimise the impact force, and a semi-passive control utilising the hysteresis stress induced phase transformations in shape memory alloys which can be tuned to reduce the amplitudes of nonlinear resonances.

EXPERIMENTAL SETUP

The experimental investigations were carried out on an impact oscillator [1], consisting of a mass supported by parallel leaf springs providing the primary stiffness and preventing the mass from rotation. In the case of the elastic oscillator, the secondary stiffness is provided by a beam mounted on a separate column. Contact between the mass and the beam occurs via an adjustable bolt. The length of the bolt can be adjusted to control the gap, g. The secondary stiffness can be varied by changing the length of the beam. In the SMA configuration, a hinged beam was used. The beam was tethered by SMA wires which provided the stiffness support. When the displacement of the beam was sufficiently high the wires underwent a stress induced phase transformation which resulted in energy dissipation and a softening nonlinearity, [2]

The oscillator rig was mounted on an electro-dynamic shaker which provided a controllable excitation through the base. Displacement of the oscillator and beam were measured with eddy current probe displacement transducers, and accelerations of the oscillator and support were measured using accelerometers. A data processing system was used which allowed phase portraits to be calculated in realtime. For the comparison of theory and experiments, a harmonic excitation of the base was used. For the impulsive control, an additional impulsive force was applied via the base for a short time, at the required instant.

IMPACT OSCILLATOR

The equation for the impact oscillator can be written in nondimensional form as [1],

$$\ddot{x} + 2\xi\dot{x} + x + \beta(x-g)H(x-g) = Asin(\omega t) \tag{1}$$

where $H(\cdot)$ is the Heaviside step function, β is the stiffness ratio, ξ is the damping and g is the gap. Figure 1 shows a range of highly nonlinear responses, and correspondence to the model of an impact oscillator. The phase portraits and Poincaré sections are in good agreement, even the fine structure of the chaotic attractors can be seen in the experimental data.

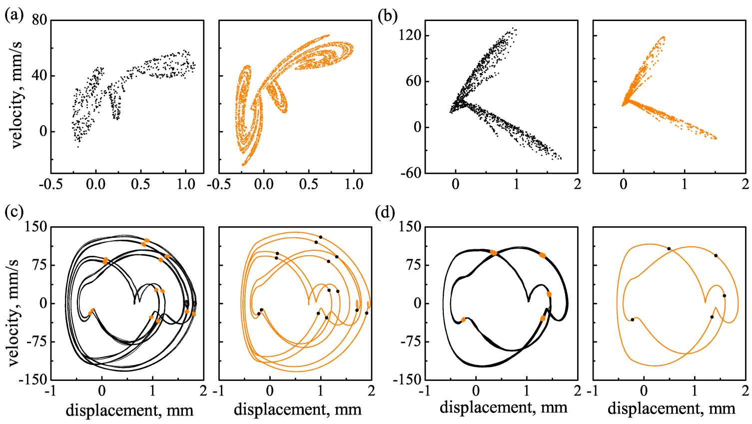

Figure 1. Phase portraits and Poincaré maps at $A = 0.2$ mm and: (a) $\omega = 2.43$, (b) $\omega = 3.41$, (c) $\omega = 3.74$ and (d) $\omega = 4.00$. Experimental data are shown in black, and numerical results for the elastic impact oscillator are in orange.

VIBRATION REDUCTION USING SMAS

Shape memory alloys have the property that they undergo phase transformations from one solid phase to another as a result of mechanical stress or temperature variation. Cyclically applied stress results in a hysteresis loop due to the different loading and unloading paths in the stress-strain plane. Since the onset of the phase transformation can be tuned

[a)] Corresponding Author: James Ing. E-mail: j.ing@abdn.ac.uk by an appropriate choice of the prestress this can be effectively used as a semi-passive control by tuning the SMA so that the hysteresis effect is only significant beyond a certain vibrational threshold. This is demonstrated in Figure 2 for two different nonlinear resonances of an impact oscillator supported by an SMA constraint. From this plot it is clear that the SMA model and the elastic impact oscillator accurately predict the response for low amplitudes. Beyond a certain threshold the elastic model overpredicts the amplitude, and hysteresis in SMA model results in a decreased amplitude closer the to experimental results. The simulations were based on a thermomechanical model of SMAs based on the internal variable model. Details of the development of the model, and the empirical parameter determination, are contained in [2] and references therein.

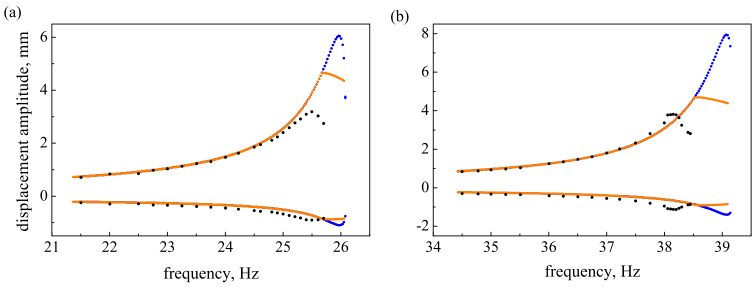

Figure 2. Resonance peak-to-peak displacement amplitudes at $A = 0.1$ mm in frequency ranges corresponding to (a) the second and (b) the third resonance peaks in a period adding cascade: experimental data (black dots), calculated values for the SMA oscillator (orange plot) and calculated amplitudes for the elastic oscillator (blue plot).

IMPULSIVE CONTROL

As was shown in [3], it is possible for multiple attractors to coexist in the impact oscillator. Due to the softening nonlinearity resulting from SMAs additional attractors may be born. Therefore ample opportunities exist for reduction of the response amplitude without modifying the system dynamics or using a costly continuous control. An intermittent perturbation to the system was applied at an instant calculated to minimise the required control effort based on comparison of the observed and desired responses in the phase plane. Further details are given in [4]. It is clear from Figure 3 that the response moves to the chosen attractor after the application of 3 closely spaced intermittent control efforts, and that this motion is subsequently stable due to the nonlinearity.

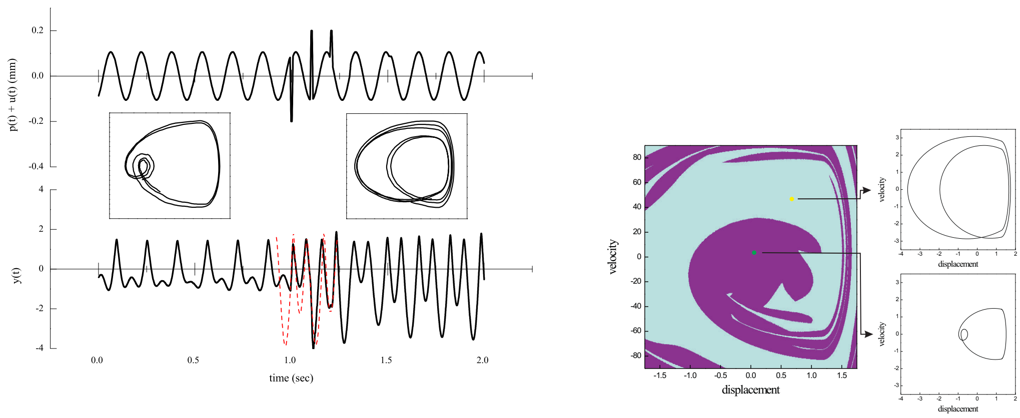

Figure 3. (a) Experimental results for intermittent control of the impact oscillator at $\xi = 0.01$, $\beta = 29$, $g = 1.26$, $\Gamma = 1.0385$, and $\omega = 0.686$. The red dash line denotes the desired displacement. Details about the control parameters are given in [4]. (b) Calculated basins of attraction for the co-existing solutions.

CONCLUSIONS

An experimental impact oscillator was introduced, and shown to be well described by a simple model. Semi-passive and intermittent controls were shown to be able to exploit the nonlinearity of the system in order to reduce the amplitude of vibration while still preserving the natural dynamical richness of the system.

References

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