NONLINEAR FORCED VIBRATIONS OF IN-PLANE TRANSLATING VISCOELASTIC PLATES WITH 3:1 INTERNAL RESONANCE

You-Qi Tang* a) & Li-Qun Chen**
*School of Mechanical Engineering, Shanghai Institute of Technology, Shanghai 201418, China
**Department of Mechanics, Shanghai University, Shanghai 200444, China

Transverse vibration of an in-plane tensioned plate moving in its plane is a significant problem that was first studied by Ulsoy and Mote [1]. Historically, the forced vibration of in-plane translating plates has been an important subject of research. However, there are only a few important publications on the forced vibration of in-plane translating plates including [2, 3, 4].

A thin rectangular plate, with mass per area ρ, Young's modulus E, Poisson's ratio μ, viscosity coefficient η, initial tension N_{x0} in the x direction only, and length a, width b, and thickness h in the x, y, and z directions respectively, lie in plane xy and moves with speed Γ in direction x dependent on time t under the distributed external surface load $F=F_0\cos(\Omega t)$ in the transverse direction. The out-plane forced vibration is governed by

$$\begin{aligned}&w,_{tt}+2\gamma w,_{xt}+\left(\gamma^2-1\right)w,_{xx}+\zeta\left(w,_{xxxx}+2\xi^2 w,_{xxyy}+\xi^4 w,_{yyyy}\right)=\varepsilon F_1\cos\omega T_0+6\varepsilon\zeta\Big[\left(w,_{xx}+\mu\xi^2 w,_{yy}\right)w,_x^2+\xi^2\left(\mu w,_{xx}\right.\\&\left.+\xi^2 w,_{yy}\right)w,_y^2+2\xi^2\left(1-\mu\right)w,_x\,w,_y\,w,_{xy}\Big]-\varepsilon\eta\Big[w,_{xxxxt}+2\xi^2 w,_{xxyyt}+\xi^4 w,_{yyyyt}+\gamma\left(w,_{xxxxx}+2\xi^2 w,_{xxxyy}+\xi^4 w,_{xyyyy}\right)\Big]+o\left(\varepsilon\right),\end{aligned} \tag{1}$$

Dimensionless transverse displacement w and other parameters are defined by

$$w\leftrightarrow\frac{w}{\sqrt{\varepsilon}h},t\leftrightarrow\frac{t}{a}\sqrt{\frac{N_{x0}}{\rho}},x\leftrightarrow\frac{x}{a},y\leftrightarrow\frac{y}{b},\zeta=\frac{D}{N_{x0}a^2},\gamma=\Gamma\sqrt{\frac{\rho}{N_{x0}}},\xi=\frac{a}{b},\eta\leftrightarrow\frac{D\eta}{\varepsilon Ea^3\sqrt{\rho N_{x0}}},\omega=a\Omega\sqrt{\frac{\rho}{N_{x0}}},F_1=\frac{\rho a^2F_0}{\varepsilon^{1.5}hN_{x0}}. \tag{2}$$

The method of multiple scales will be employed to Eq. (1). One assumes an expansion of displacement in the form

$$w(x,y,t;\varepsilon)=w_0\left(x,y,T_0,T_1\right)+\varepsilon w_1\left(x,y,T_0,T_1\right)+O\left(\varepsilon^2\right), \tag{3}$$

Substitution of Eq. (3) into (1) and then equalization of coefficients of ε^0 and ε^1 in the resulting equations lead to

$$\varepsilon^0:\quad w_0,_{T_0T_0}+2\gamma w_0,_{xT_0}+\left(\gamma^2-1\right)w_0,_{xx}+\zeta\left(w_0,_{xxxx}+2\xi^2 w_0,_{xxyy}+\xi^4 w_0,_{yyyy}\right)=0, \tag{4}$$

$$\begin{aligned}\varepsilon^1:\ &w_1,_{T_0T_0}+2\gamma w_1,_{xT_0}+\left(\gamma^2-1\right)w_1,_{xx}+\zeta\left(w_1,_{xxxx}+2\xi^2 w_1,_{xxyy}+\xi^4 w_1,_{yyyy}\right)=\\&-\left\{\eta\left[w_0,_{xxxxT_0}+2\xi^2 w_0,_{xxyyT_0}+\xi^4 w_0,_{yyyyT_0}+\gamma\left(w_0,_{xxxxx}+2\xi^2 w_0,_{xxxyy}+\xi^4 w_0,_{xyyyy}\right)\right]+2\left(w_0,_{T_0T_1}+\gamma w_0,_{xT_1}\right)\right\}\\&+6\zeta\left[\left(w_0,_{xx}+\xi^2\mu w_0,_{yy}\right)w_0,_x^2+\xi^2\left(\mu w_0,_{xx}+\xi^2 w_0,_{yy}\right)w_0,_y^2+2\xi^2\left(1-\mu\right)w_0,_x\,w_0,_y\,w_0,_{xy}\right]+F_1\cos\omega T_0,\end{aligned} \tag{5}$$

We describe the nearness of $\omega_{s'l'}$ to $3\omega_{sl}$ and ω to $\omega_{s'l'}$ using the detuning parameters σ_1 and σ_2. For this, we write the frequency relations for the first primary resonance and the 3:1 internal resonance as

$$\omega_{s'l'}=3\omega_{sl}+\varepsilon\sigma_1,\ \ \omega=\omega_{sl}+\varepsilon\sigma_2. \tag{6}$$

where ω_{sl} (sl=1, 2, 3, 4...) and $\omega_{s'l'}$ ($s'l'$=2, 3, 4...) are, respectively, the slth and the $s'l'$th frequencies of the ε^0-order system. The solution to Eq. (4) can be assumed in the form

$$w_0\left(x,y,T_0,T_1\right)=\psi_{sl}\left(x,y\right)A_{sl}\left(T_1\right)\mathrm{e}^{\mathrm{i}\omega_{sl}T_0}+\psi_{s'l'}\left(x,y\right)A_{s'l'}\left(T_1\right)\mathrm{e}^{\mathrm{i}\omega_{s'l'}T_0}+cc, \tag{7}$$

Substitution of Eqs. (6) and (7) into (5), the solvability condition demands the orthogonal relationships

$$\begin{aligned}&A_{sl},_{T_1}+\eta m_1A_{sl}+g_{11}A_{sl}^2\bar{A}_{sl}+g_{12}A_{sl}A_{s'l'}\bar{A}_{s'l'}+h_1A_{s'l'}\bar{A}_{sl}^2\,\mathrm{e}^{\mathrm{i}\sigma_1T_1}+nF_1\,\mathrm{e}^{\mathrm{i}\sigma_2T_1}=0,\\&A_{s'l'},_{T_1}+\eta m_2A_{s'l'}+g_{22}A_{s'l'}^2\bar{A}_{s'l'}+g_{21}A_{s'l'}A_{sl}\bar{A}_{sl}+h_2A_{sl}^3\,\mathrm{e}^{-\mathrm{i}\sigma_1T_1}=0.\end{aligned} \tag{8}$$

Numerical tests strongly indicate that m_1 and m_2 are positive real numbers, g_{11}, g_{12}, g_{21}, and g_{22} are pure imaginary numbers with negative imaginary parts, while h_1 and h_2 are complex numbers, and these will be assumed as true in the following. Express the solution to Eq. (8) in polar form as follows

$$A_{sl}=\alpha_{sl}\left(T_1\right)\mathrm{e}^{\mathrm{i}\beta_{sl}(T_1)},\quad A_{s'l'}=\alpha_{s'l'}\left(T_1\right)\mathrm{e}^{\mathrm{i}\beta_{s'l'}(T_1)}. \tag{9}$$

Substituting Eq. (9) into (8) and integrating the results, the amplitude α_{sl} and $\alpha_{s'l'}$ and the new phase angles $\theta_1=\sigma_1T_1-3\beta_{sl}+\beta_{s'l'}$ and $\theta_2=\sigma_2T_1-\beta_{s'l'}$ should be constant for steady-state solutions. Then, we obtain

$$9\eta^2m_1^2+\left[\sigma_1+\sigma_2+3\left(g_{11}^1\alpha_{sl}^2+g_{12}^1\alpha_{s'l'}^2\right)\right]^2=9C^2\alpha_{sl}^2\alpha_{s'l'}^2\left|h_2\right|^2, \tag{11}$$

$$9\eta^2\left(Cm_2\alpha_{s'l'}^2-m_1\alpha_{sl}^2\right)^2+\left\{3C\alpha_{s'l'}^2\left(\sigma_2+g_{21}^1\alpha_{sl}^2+g_{22}^1\alpha_{s'l'}^2\right)+\alpha_{sl}^2\left[\sigma_1+\sigma_2+3\left(g_{11}^1\alpha_{sl}^2+g_{12}^1\alpha_{s'l'}^2\right)\right]\right\}^2=9C^2\alpha_{s'l'}^2\left|n\right|^2F_1^2. \tag{12}$$

The stability of the steady state equations can be investigated by the Lyapunov linearized theory. One has to consider the amplitude and phase modulation equations and construct the associated Jacobian matrix evaluated at the fixed points. The eigenvalues of the associated Jacobian matrix should have negative real parts to maintain stability.

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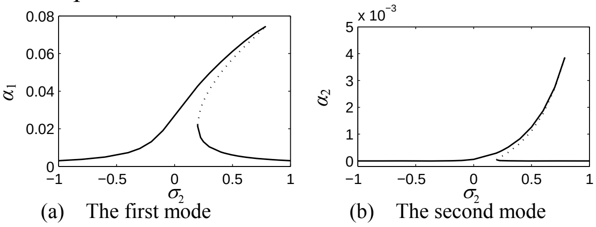
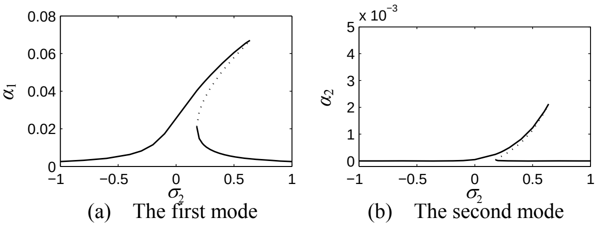
a) You-Qi Tang. Email: tangyouqi2000@163.com.

Consider an in-plane translating viscoelastic plate with four edges simple supports at both ends. Here, we choose the basic parameters ζ=1, ξ=1, η=0.0001, F_1=0.35, and γ=3.9243. When γ=3.9243, ω_1=13.95966, ω_2=43.8036, and σ_1=1.92462. When γ=4.08823, ω_1=13.4318, ω_2=43.27816, and σ_1=2.98276. The solid lines and dots represent the stable and unstable responses, respectively. Fig. 1-4 shows the frequency-amplitude curves in the first two modes for different parameters.



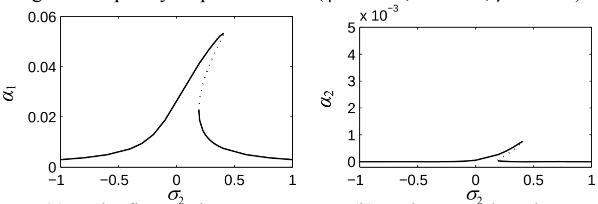
(a) The first mode (b) The second mode

Fig.1 Frequency response curves (η=0.0001, F_1=0.35, γ=3.9243)



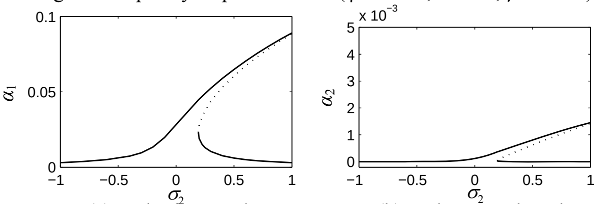
(a) The first mode (b) The second mode

Fig.2 Frequency response curves (η=0.0001, F_1=0.3, γ=3.9243)



(a) The first mode (b) The second mode

Fig.3 Frequency response curves (η=0.00015, F_1=0.35, γ=3.9243)



(a) The first mode (b) The second mode

Fig.4 Frequency response curves (η=0.0001, F_1=0.35, γ=3.4)

The characterization parameter ε is no specific physical meaning. In the following numerical investigations, let ε=1. The first four-order dimensionless frequencies of the linear generating system have been confirmed by Tang and Chen [5]. In this paper, the same forms of the sampling points are chosen. The DQ analogue of the full non-autonomous system for the one-dimensional reduced plate model can be obtained as (i=2, 3,…,N_x-1)

$$\begin{aligned}\ddot{\phi}_i = & -2\gamma\sum_{k=2}^{N_x-1} A_{ik}^{(1)}\dot{\phi}_k - \eta\left(\sum_{k=2}^{N_x-1}\tilde{A}_{ik}^{(4)}\dot{\phi}_k - 2m^2\pi^2\xi^2\sum_{k=2}^{N_x-1}\tilde{A}_{ik}^{(2)}\dot{\phi}_k + m^4\pi^4\xi^4\dot{\phi}_i\right) - \left(\gamma^2-1\right)\sum_{k=2}^{N_x-1}\tilde{A}_{ik}^{(2)}\phi_k - \zeta\left(\sum_{k=2}^{N_x-1}\tilde{A}_{ik}^{(4)}\phi_k - 2m^2\pi^2\xi^2\right.\\ & \left.\sum_{k=2}^{N_x-1}\tilde{A}_{ik}^{(2)}\phi_k + m^4\pi^4\xi^4\phi_i\right) + \frac{3}{2}\zeta\left[3\left(\sum_{k=2}^{N_x-1}\tilde{A}_{ik}^{(2)}\phi_k - \mu m^2\pi^2\xi^2\phi_i\right)\left(\sum_{k=2}^{N_x-1}A_{ik}^{(1)}\phi_k\right)^2 + m^2\pi^2\xi^2\left(\mu\sum_{k=2}^{N_x-1}\tilde{A}_{ik}^{(2)}\phi_k - m^2\pi^2\xi^2\phi_i\right)\phi_i^2 + \right.\\ & \left. 2m^2\pi^2\xi^2\left(1-\mu\right)\left(\sum_{k=2}^{N_x-1}A_{ik}^{(1)}\phi_k\right)^2\phi_i\right] - \eta\gamma\left(\sum_{k=2}^{N_x-1}\tilde{A}_{ik}^{(5)}\phi_k - 2m^2\pi^2\xi^2\sum_{k=2}^{N_x-1}\tilde{A}_{ik}^{(3)}\phi_k + m^4\pi^4\xi^4\sum_{k=2}^{N_x-1}A_{ik}^{(1)}\phi_k\right) + \frac{4}{\pi}F_1\cos\omega t.\end{aligned} \tag{13}$$

Here, we choose the sampling point N_x=9. W_0 is the max value of w_0. The heavy lines, the dash-dotted lines, and the fine lines denote the stable analytical results; the dotted lines and the dashed lines denote the unstable analytical results; the symbols '*', the symbols 'Δ', and the symbols '◊' denote the numerical results. Fig. 5-7 presents the comparison of the analytical and numerical results for different viscosity coefficients, excitation amplitudes, and in-plane translating speeds. It is observed that the smaller viscosity coefficient, the larger excitation amplitude, and the larger in-plane translating speed leads to the larger stable steady-state response amplitudes.

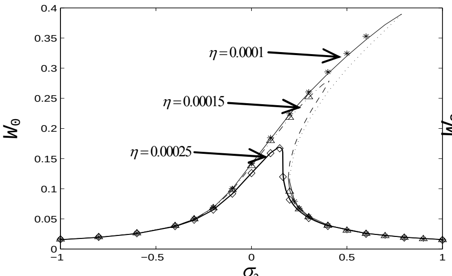


Fig.5 Comparison of analytical and numerical results for different viscosity coefficients η

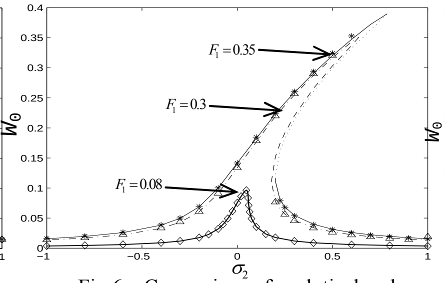


Fig.6 Comparison of analytical and numerical results for different excitation amplitudes F_1

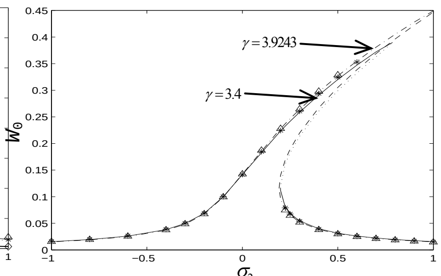


Fig.7 Comparison of analytical and numerical results for different in-plane translating speeds γ

References

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