MAGNETIC FIELD DRIVEN MICRODEVICES

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Summary The properties of a magnetic field driven microdevice with a permanent magnet attached to the end of a flexible filament are investigated. It is shown that the magnet on average orientates perpendicularly to the field and displays self-propulsive behavior.

MODEL OF A PERMANENT MAGNET WITH A FLEXIBLE TAIL AND ITS NUMERICAL SIMULATION

Magnetic field driven self-propelling microdevices have attracted great interest recently [1, 2, 3, 4]. Energy supply to these devices is due to the external AC magnetic field. Although there is a view that these devices should not be classified as self-propelling due to the external energy supply [5] nevertheless the underlying physics is the same as in models which mimic, for example, the self-propulsion of filaments with internal torques. The behavior of these devices is similar to different living organisms as well. The motion of a superparamagetic filament with broken spatial symmetry in an AC field mimics the motion of spermatozoa [1, 2, 3, 4], while the back and forth motion of a ferromagnetic filament in an AC magnetic field [6, 7] is similar to the self-propulsion of biflagellate organisms [8].
It is evident that the application of the magnetic field enables the creation of different self-propelling mechanisms and the question is which should be chosen as the more efficient. A new configuration of the magnetic swimmer was recently proposed as a candidate for a more efficient swimmer [5], it consists of a flexible filament with a permanent magnet attached at one of its ends. A comparison of this device with other known swimmer configurations shows that it provides a higher velocity of self-propulsion. Here we analyze the properties of this interesting swimmer.
Let us consider an inextensible filament with a length L and a bending modulus A. A permanent magnet with a magnetic moment m is attached at its end. The direction of the tangent vector of the center-line of the filament at this end is clamped to the direction of the magnetic moment. The force $\vec{F}$ and the torque $\vec{M}$ in the cross-section of the filament represented by $\vec{r} = \vec{r}(l)$ are given by the Kirchhoff model of the elastic rod

$$\vec{F} = -A\frac{d^3\vec{r}}{dl^3} + \Lambda\vec{t}\,; \vec{M} = A\frac{d\vec{r}}{dl} \times \frac{d^2\vec{r}}{dl^2}\,. \tag{1}$$

The equation of motion is given in the Rouse approximation $B^{-1}\vec{v} = d\vec{F}/dl$, where B is the mobility matrix. Boundary conditions at the ends of the filament as follow from the torque balance equation $d\vec{M}/dl + \vec{t} \times \vec{F} + \vec{K} = 0$, where $\vec{K}$ is the torque due to the applied magnetic field $\vec{m} \times \vec{H}/b$ (b is the length of permanent magnet) read $Ad^2\vec{r}/dl^2\mid_{l=L}= m(\vec{H} - \vec{t}\vec{t}\cdot\vec{H})$ and $d^2\vec{r}/dl^2\mid_{l=0}= 0$ on the other free end. According to the algorithm based on the introduction of the projection operator on the space of configurations which satisfy the inextensibility constraint the two remaining necessary boundary conditions are formulated as equations of motion of the ends with finite hydrodynamic drag coefficients as described elsewhere [7]. Length is scaled by L (the length of the filament), time by the elastic relaxation time $\tau_e = \zeta_\perp L^4/A$. As a result the behavior of the device is determined by two dimensionless parameters,- the magnetoelastic number $Cm = mHL/A$ and the ratio of the length of the filament to the elastic penetration length $L_e = (A/\zeta_\perp\omega)^{1/4}$.
The first question which arises is how the swimming direction of the device is connected with the applied field. An insight to this problem is given by the fact that a flexible ferromagnetic rod in an AC field with sufficiently high frequency orientates perpendicularly to the field (see [7]). Here we illustrate the fact that a permanent magnet with an attached flexible filament on average orientates perpendicularly to the applied field. There are two time scales in the problem, - a fast time scale determined by the period of the AC field and a slow time scale related to the change of the mean orientation of the filament. The angle $\delta\vec{\varphi}$ of the tangent vector fast oscillation in an AC field is introduced according to $\delta\vec{t} = \delta\vec{\varphi} \times \vec{t}_0$. The configuration of the filament is represented as $\vec{r}_0 + \vec{r}' + \vec{w}$, where $\vec{r}_0 = \vec{t}_0 l$ is the mean actual orientation of the filament and $\vec{r}'$ is the oscillating part of the filament deformation and $\vec{w}$ is a slowly changing deviation from the straight configuration $\vec{r}_0(l)$ the variation of which within a period of the AC field is neglected. The equation of motion for the mean configuration then is as follows

$$\zeta_\perp \vec{r}_{0,t} = -A\vec{w}_{,llll}\,. \tag{2}$$

The condition of solvability of Eq.(2) gives

$$\zeta_\perp \frac{2L^3}{3}\vec{\omega} = -<\delta\vec{\varphi}\vec{H}\cdot\vec{t}_0> \;, \tag{3}$$

where $\vec{r_0}_{,t} = \vec{\omega} \times \vec{t}_0$ and $<>$ denotes the time average per period of the AC field. The oscillating $\delta\vec{\varphi}$ in the case of the field $\vec{H} = H(\cos(\omega t), \sin(\omega t), 0)$ rotating in the (x, y) plane is found by the solution of the linear problem $i\omega\zeta_\perp f = -Af_{,llll}; Af_{,l}(L) = 1; f_{,l}(0) = 0; f_{,ll}(0) = f_{,ll}(L) = 0$ in the form ($f(L) = k_1 - ik_2$ ($k_1 > 0$)):

$$\delta\vec{\varphi} = -\frac{mH}{A}\vec{e}_x \times \vec{t}_0\Re\Big(\exp(i\omega t)f(l)\Big) - \frac{mH}{A}\vec{e}_x \times \vec{t}_0\Im\Big(\exp(i\omega t)f(l)\Big)\,. \tag{4}$$

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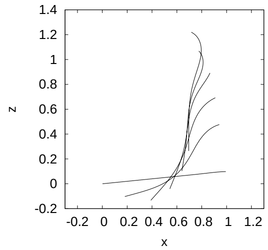

Figure 1. $Cm = 20; L/L_e = 7.5$

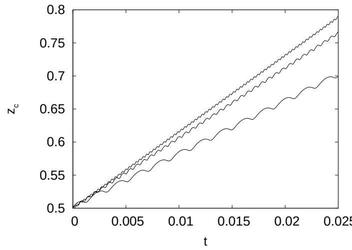

Figure 2. $Cm = 10; L/L_e = 6.3; 8.3; 9.9$

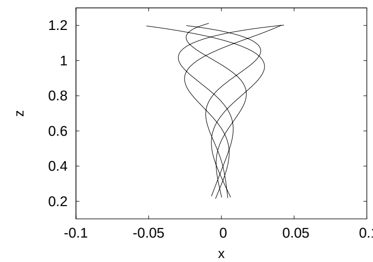

Figure 3. $Cm = 10; L/L_e = 8.3$

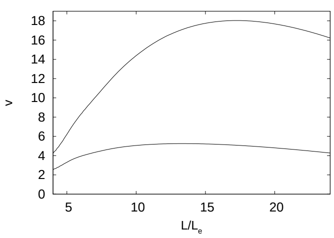

Figure 4. $Cm = 5; 10$

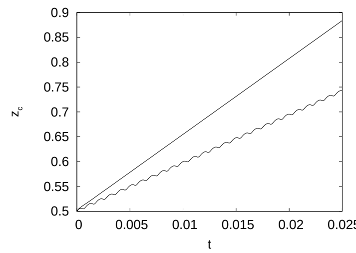

Figure 5. $Cm = 10; L/L_e = 7.5$

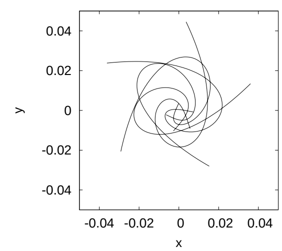

Figure 6. $Cm = 10; L/L_e = 7.5$

As a result the equation for the mean orientation reads

$$\zeta_\perp \frac{2L^3}{3}\vec{\omega} = -\Big[\vec{t}_0 \times \frac{\partial}{\partial \vec{t}_0}\Big]\Big(-\frac{k_1(mH)^2}{4A}(\vec{e}_z \cdot \vec{t}_0)^2\Big)\;. \tag{5}$$

From Eq.(5) it follows that a permanent magnet with an attached flexible filament orientates perpendicularly to the plane of the rotating field. This is confirmed by the numerical experiment,- a filament initially confined to the (x, z) plane and oriented in a direction close to that of the field $\vec{H} = H(\cos(\omega t), 0, 0)$ displays oscillatory behavior and appears to slowly converge to a configuration with the mean orientation perpendicular to the field (Fig.1). The filament is swimming in the direction perpendicular to the applied field due to broken spatial symmetry. The dependence of its center of mass coordinate on time for several values of the parameter L/L_e is shown in Fig.2. The swimming is due to the bending deformations propagating from the head to the tail as shown in Fig.3. The swimming consists of forth and back stages as it is characteristic of the ferromagnetic swimmers [6, 7] and living organisms [8]. This swimmer swims faster at similar values of the parameters than the ferromagnetic swimmer [6, 7]. The dependence of the velocity of self-propulsion on L/L_e for two values of Cm is shown in Fig.4. It is possible to compare the swimming speeds for unidirectional and rotating fields for the same values of the parameters (Fig.5). Thus we may conclude that from the point of view of maximizing the swimming velocity the application of the rotating field is more advantageous. Configurations of the filament in the projection on the plane of the rotating field for one period are shown in Fig.6. We see that the propulsion is due to the propagating bending deformations and is not due to the chirality of the filament.

CONCLUSIONS

In conclusion permanent magnet with a flexible tail orientates perpendicularly to the AC magnetic field. Velocity of its self-propulsion depends on the field strength and its frequency. Rotating field is more efficient than unidirectional AC field for driving the micro-device.

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