Simulation of non-breaking and breaking solitary wave run-up and impact by SPH method

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Summary The smoothed particle hydrodynamics (SPH) method is realised to simulate the nonbreaking and breaking solitary wave run-up and impact on the vertical wall. The second order accuracy is achieved by introducing the correction step to improve the traditional SPH method. SPH-computed wave profiles are in good agreement with the experiment results for both nonbreaking and breaking solitary wave, and more accurate simulate run-up for breaking solitary wave as compared with the Eulerian Shallow-Water Equation (ESWE) model.

INTRODUCTION

The tsunami generated by the Great East Japan Earthquake in 2011 caused tremendous damages to residential and industrial installations at the affected area. The run-up of the tsunami has damaged the power cooling units of the Fukushima nuclear power plant, resulting in the radionuclide spill. All of these, including the lessons learnt from the 2004 Indian Ocean Tsunami disaster, have shown that in spite of studies during past decades the tsunami damage still underestimated.

In this study, the enhanced SPH method [1] is used to study the solitary wave run-up and break over a sloping beach and impact on a vertical wall. The simulation results are compared with that of the famous tsunami model, Tunami-N2 [2][3], and verified against the experiments conducted at the U.S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi [4].

SPH METHODOLOGY

The SPH method was first introduced by Gingold and Monaghan [5]. The expression has the form of

$$f(\mathbf{x}) = \int_{\Omega} f(\mathbf{x}')W(\mathbf{x}-\mathbf{x}',h)d\mathbf{x}' \tag{1}$$

where $W(\mathbf{x}-\mathbf{x}',h)$ is the kernel function. The SPH approximation of the Navier-Stokes equations can be approximated in the following forms

$$\frac{d\rho_i}{dt} = -\rho_i \nabla \cdot \mathbf{u} = -\rho_i \sum_j \frac{m_j}{\rho_j}\left(\mathbf{u}_j - \mathbf{u}_i\right)\cdot \nabla_i W_{ij} \tag{2}$$

$$\frac{d\mathbf{u}_i}{dt} = -\sum_j m_j \left(\frac{p_i}{\rho_i \rho_j} + \frac{p_j}{\rho_i \rho_j} + \Pi_{ij}\right)\nabla_i W_{ij} + \mathbf{g}_i \tag{3}$$

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{u}_j \tag{4}$$

in Eq. (4) the term Π_{ij} is artificial viscosity.

SOLITARY WAVE RUN-UP SIMULATIONS AND DISCUSSIONS

The experiment was conducted at the U.S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi by Ward [5]. The configuration of the experiment is show in (a) of Figure 1. In the experiment, three different solitary waves are generated by the movement of the piston paddle. The target wave height, h_{tar}/d, of each case are 0.05 (case A), 0.3 (case B) and 0.7 (case C), respectively. The flume is discretized by about 1.1 million uniform diameter particles with size dx = 0.002m and simulated in 100 parallel processors. The comparisons of surface elevation sequential snapshots for SPH simulation, Tunami-N2 simulation and experiment data are shown in (b), (c) and (d) of Figure 2 . The surface elevations of SPH simulations for all of the three cases, in particular after the wave breaking, agree quite well with the experiment. Figure 2 shows the snapshots of wave propagation and impact on the wall for case C. Figure 3 shows pressure time history at the still water level on the wall for the three cases. In case B and case C, sharp rises of pressures can be observed, which is characterized by the impact of the wave front on the wall. The impact pressure of case C could exceed several times the static pressure. Fail to simulate the subsequent process of wave breaking, as it usually happens in non-breaking tsunami models, may lead to under-predictions of the water velocity under the wave as well as the run-up height, and hence under-prediction of the impact pressure.

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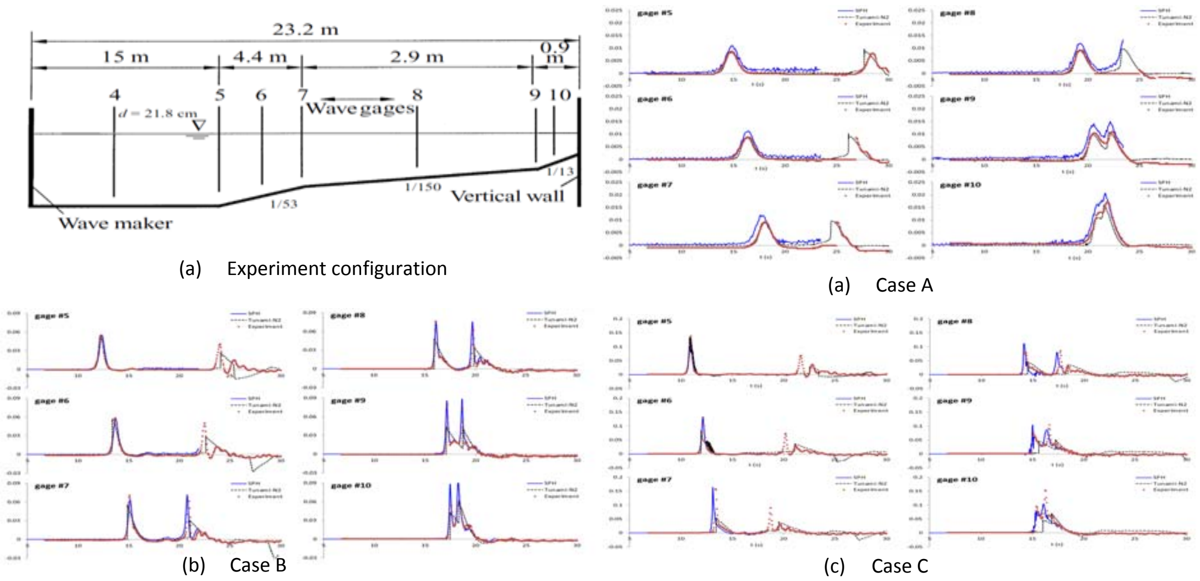

Figure 1 Experiment setup (a) and time histories of surface elevation for experiment and numerical simulation at gage 5 to gage 10 for case A (b), case B (c) and case C (d)

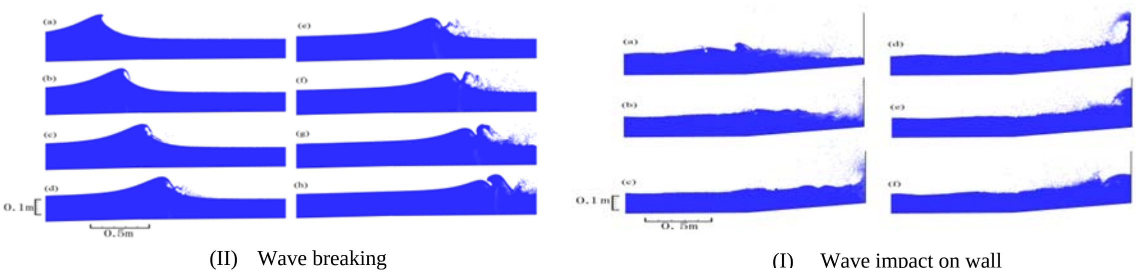

Figure 2 Snapshots of wave breaking (I) and impact on the wall (II) for case C.

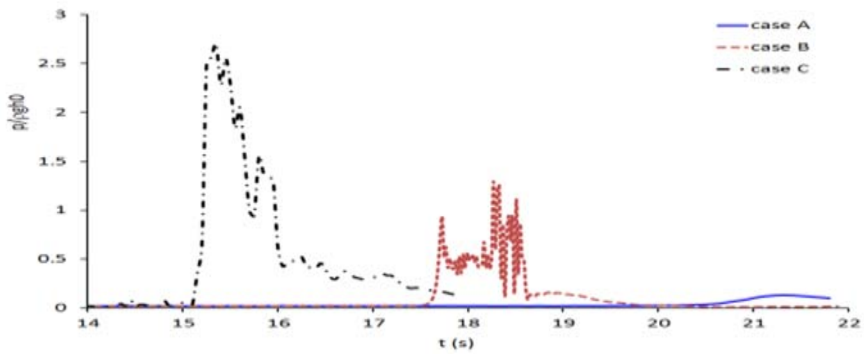

Figure 3 Impact pressure on the wall for case A, B and C at the still water level.

CONCLUSION

In this study, three cases of solitary wave propagation and run-up are simulated by the enhanced SPH model and compared with the Tunami-N2 numerical models and the experiment data. Snapshots are extracted from the SPH simulations for the wave propagation stage and during impact on the vertical wall. The results show the SPH method has the capability to simulate the propagation, wave breaking and wave impact on a wall.

References

[1] Xu, H., Dao, M.H., Chan, E.S.: An SPH model with C1 consistency, in: Rogers, B.D. (Ed.), 5th Internetional SPHERIC Workshop, Manchester, 2010.

[2] Goto, C., Ogawa, Y., Shuto, N., Imamura, F.: Numerical method of tsunami simulation with the leap-frog scheme (IUGG/IOC Time Project), IOC Manual,UNESCO, No. 35, 1997.

[3] Tkalich P., Dao M.H.: Tsunami Modelling and Forecasting Techniques, In book (Eds. K Moffatt & E. Shuckburgh) ENVIRONMENTAL HAZARDS. The Fluid Dynamics and Geophysics of Extreme Events, Lecture Notes Series, Institute for Mathematical Sciences, National University of Singapore - Vol. 21, World Scientific, 2011, ISBN: 978-981-4313-28-5, 332 pp, 2011.

[4] Gingold, R.A., Monaghan, J.J.: Smoothed particle hydrodynamics: theory and application to non-spherical stars. Monthly Notices of the Royal Astronomical Society 181, 375-389, 1977.

[5] Ward, D.L.: Physical model study of revere beach, massachusetts, United States, p. 71p, 1995.