TOY MODELS ILLUSTRATING COMPLEX MECHANICAL PHENOMENA

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Summary Certain mechanical toys can be used in lecture demonstrations to illustrate mechanical principles of great generality; this can bring the subject vividly to life for students, and can be of great value as a supplement to the more formal development of theoretical concepts. Three examples will be discussed: Euler's disc, a prototype of the concept of 'finite-time singularity' and its resolution; the 'rising egg phenomenon' as an example of 'slow instability' ; and the rattleback (or 'celt'), a prototype of symmetry-breaking instabilty and chiral behaviour. A new mechanical model will be presented for which the chiral behaviour is induced by spin of the rattleback (rather than by its unperturbed static geometry).

INTRODUCTION

A famous photograph of two Nobel Laureates, Niels Bohr and Wolfgang Pauli, intrigued by the behaviour of the toy known as the 'tippe-top' (figure 1), may provide sufficient justification for the claim that mechanical toys can provoke curiosity leading to new physical insight. In fact, it is in the context of spin dynamics where the dominant effects of inertia are moderated by weak frictional forces that one's intuition is most challenged; toys that exhibit counter-intuitive behaviour can serve a valuable purpose in the classroom, in exciting student curiosity and in motivating the development of appropriate mathematical description.

Three examples deserve attention, in that they provide prototypes of behaviour that appears in much more complex physical situations, and particularly in high-Reynolds-number fluid dynamics, in which again the dominant force of inertia is moderated by friction through the relatively weak viscosity of the fluid.

THE EULER DISC

The first example is the Euler disc, a disc that rolls on its rim on a smooth horizontal table, its normal being nearly vertical. Weak friction is unavoidable, so that the angle between the plane of the disc and the table decreases to zero. What is interesting is that this decrease is not exponential (as for, say, a damped pendulum), but 'catastrophic' in the sense that it reaches zero at a finite time: it is in fact a 'finite-time singularity. Discussion of this phenomenon can provide valuable motivation for the introduction of the famous unsolved finite-time singularity problem of high-Re fluid mechanics.

THE RISING EGG

The second example is the 'rising egg' phenomenon, akin to the tippe-top, but more complicated (the tippe-top has a spherical base, a great simplification in the analysis). When a hard-boiled egg is spun sufficiently rapidly on a table, its axis being horizontal, it will rise to the vertical, as a result of the frictional force acting at the point of slipping contact between the egg and the table. This illustrates a 'slow instability' above a 'critical spin rate', analogous to the instability of Poiseuille flow in a channel, similarly a threshold phenomenon driven by weak viscous friction near the

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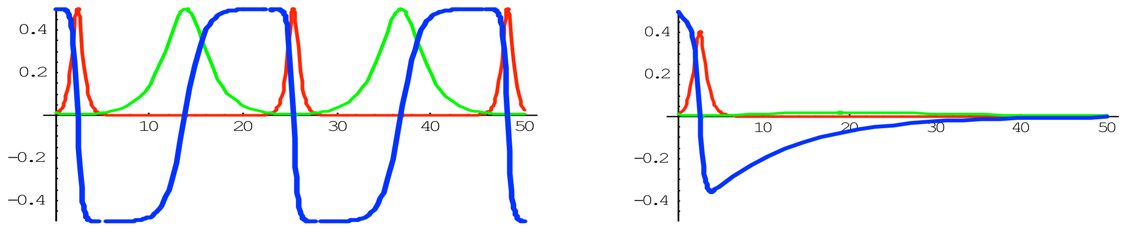
boundary. The transition from spin of the egg in the horizontal to the vertical orientation can be regarded as a simple analogue of transition from laminar to turbulent flow in the channel flow context.

THE RATTLEBACK

The third example, on which this presentation will be mainly focused, is that of the celt, or 'rattleback', a canoe-shaped object which exhibits spin asymmetry: it spins quite smoothly in one sense (say, anti-clockwise) on a table, but if spun in the opposite sense, it is subject to a 'pitching instability' which extracts energy from the rotation to such an extent that the sense of rotation reverses. The behaviour is to be sought in a very slight geometrical asymmetry -- actually a lack of mirror symmetry -- associated with rotation of the horizontal axes of inertia relative to the principal axes of curvature at the point of contact with the table.



(i) Reversals of the rattleback (ii)

A mathematical explanation of the rattleback reversals has been provided by Moffatt & Tokieda (2008). The blue curve in the figures above shows the angular velocity N about the vertical (i) when dissipative effects are neglected and the motion is periodic, and (ii) (more realistically) when weak empirical friction parameters are included in the model.The red curve shows the energy in the strong pitching instability excited when $N > 0$. The green curve shows the energy in a weaker 'rolling' instability excited when $N < 0$. Both instabilities may be observed in practice.

When the rattleback is flexible and such that its geometry can adjust in response to centrifugal forces, the dynamics is significantly altered, and the reversals are symmetrised; a modified toy exhibiting this behaviour will be demonstrated. Here, the toy provides a simple model for rotational generation of helicity, turbulent dynamo action, and reversals of the Earth's magnetic field.

CONCLUSION

For teaching purposes, simple mechanical toys can be effectively used to 'bring to life' the subtleties of highly complex natural phenomena.

References

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