

TEACHING CHAOS FOR FRESHMEN AS A LIBERAL EDUCATION COURSE

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Summary “Chaos and Nonlinear Thinking” is a liberal education course for freshmen of all three categories, namely, science and technology, economics and management, as well as humanities, in Shanghai University. The course is devoted to the scientific concept of chaos and its cultural impacts. Nonlinearity is treated not only as a mathematical model but also a thinking mode. The course helps the students to understand the diversity, the uncertainty, and the unpredictability in the real world.

INTRODUCTION

Chaos theory emerges in the last four decades. Its impacts are beyond the physical sciences or even natural sciences. Chaos theory provides engineers, social scientists and humanists with new views and tools. Some of the vocabulary such as chaos and nonlinearity has evolved from the specific terminology to a part of common language of the publics. Therefore, chaos theory has become a part of general human knowledge.

In the fall of 2011, all freshmen in Shanghai University no longer settle a definite field such as physics or mechanical engineering as they enrol in the university. Instead, they belong to one of three categories, science and technology, economics and management, and humanities and choose the fields in the second year. New liberal education courses are designed to satisfy the various needs of the freshmen. “Chaos and Nonlinear Thinking” is such a course.

The following presentation will account for the pedagogical features and technical contents of Chaos and Nonlinear Thinking. Its course’s details and the students’ feedback are also briefed.

PEDAGOGICAL FEATURES

The course is accessible for students of science and technology, economics and management, and humanities. Therefore, no much mathematical or physical sophistication is assumed. Actually, most of the students are studying calculus and university physics that are helpful but not necessary.

As its prerequisites are rather limited, the course is taught in an informal style. Many pictures are adopted to visualize mathematical arguments and scientific concepts and to rouse students’ interests. Mathematics is kept in the minimum, while necessary equations are used to clarify the exactness and the soundness of the knowledge.

The course is essentially interdisciplinary among sciences, engineering, and humanists. It provides the students with a broad version beyond mathematics and science. In fact, the course is involved in not only engineering applications but also history, sociology and methodology of science. Some impacts of chaos theory on philosophy, aesthetics and literature are also concerned.

TECHNICAL CONTENTS

The total course consists of 8 lectures.

Lecture 1 is an introductory description of the concept of nonlinearity that the ratio of input and output is not constant. Meanwhile, the mathematical definition of linearity is also presented, and the significances and the limits of linear models are discussed.

Lecture 2 focuses on the logistic map. It begins with the derivation of the map as a model of biological populations with nonoverlapping generations. Then period-1, period-2, and period-4 point are located, and their stabilities are analyzed. Chaos in the map is introduced with the properties of recurrence without periods, sensitiveness to initial states, as well as bifurcation diagram with self-similarity and periodic windows.

Lecture 3 elucidates the conceptual evolution of chaos with the culture background, the historical developments, and an illustrative example. The word “chaos” has been found in western classics such as *The Theogony* of Hesiod (translated by Hume G. Evelyn-White), *The Metamorphoses* of Ovid (translated by Horace Gregory), Holy Bible (King James Version), as well as Chinese classics such as Zhuang Zi. Some old sayings regarding the importance of initial steps have been collected. The rudiment of chaos in the sense of nonlinear dynamics can be traced back to James Clerk Maxwell’s Cambridge speech referring instability in 1873, Jacques Hadamard’s paper on the geodesic flow on a surface of negative curvature in 1898, and Pierre Duhem’s interpretation of Hadamard’s idea in 1906. Then Henri Poincaré founded chaos theory mathematically by discovering transverse homoclinic points in 1890 and conceptually by revealing chance due to the sensitiveness to initial values. The contributions of other pioneers such as Steve Smale, Edward Norton Lorenz,

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Yoshisuke Ueda, Tien-Yien Li and James A. Yorke, are presented. The lecture ends with the demonstration of chaotic motion of a forced mass-(nonlinear) spring oscillator, which was first studied by Ueda, via the time histories, the phase trajectories, and the Poincaré map.

Lecture 4 deals with the geometrical structure of chaos. Chaos is represented by a trajectory that never closes and repeats and the trajectory locates in a bounded region due to the recurrence of the motion. The Poincaré map of chaos is a set of infinite points that do not fill any loops or tori. Fractal dimension is introduced with some classic examples such as the Cantor set, the Koch curve, the Koch snow, the Sierpinski triangle and the Sierpinski carpet. The relations between the fractals and the self-similarity as well as the fractals and the sensitive dependences are discussed.

Lecture 5 is concerned with the routs to chaos. The emerging process of chaos is investigated with the variation of a system parameter. Period-doubling cascade with the Feigenbaum constants, intermittency, quasiperiodic torus breakdown, and crisis are presented with some examples.

Lecture 6 is devoted to demonstrating the ubiquity of chaos. There are examples with the backgrounds of physics, chemistry, biology, astronomy, engineering, sociology, and economics. The influences of chaos on philosophy, aesthetics and literature are briefly commented.

Lecture 7 treats numerical experiments, a striking methodological character of chaos theory. The general procedure and features of numerical experiments are summary through some case studies. Its importance and limits are expounded. The relation between theories and numerics are discussed.

Lecture 8 concludes the course with the tentative expatiation on nonlinear thinking. The linear view of the world is based on two assumptions that the whole is the sum of its parts and the production is proportionate to the investment. The two assumptions somehow have become a part of common sense. Nonlinear thinking overthrows the linear view. It emphasizes the diversity, the uncertainty, and the unpredictability.

COURSE'S DETAILS

The course lasts ten weeks, 2 class periods once a week. Each lecture needs 2 class periods except that lecture 3 needs 6 class periods.

The students in the class are required to read one of references [1-6] as they choose, at least some part of one. They are also recommended to read some historically significant but not too technically difficult papers, such as [7-12].

In the eighth week, all students should submit either a reading report or a project report, or both if they like. The reading report surveys some materials related to chaos, for example, a reference book or a chapter or even a section of it, a paper. Students select the reading materials themselves. The project report discusses the possible applications of chaos to the fields in which the students are going to major.

At the end of the quarter, all students should submit a sum-up report less than 1000 Chinese characters to summarize their acquirements in the subject and to give some suggestions to the instructor if applicable.

STUDENDTS' FEEDBACK

Most of students found the course novel and stimulating and experienced the happiness of pursuing new knowledge. Even so, some of them, especially those majoring in humanities, felt it too abstruse or even too obscure. The historical stories and the popular images are attractive, but mathematical terms and operations are hard to understand. A few students are very interested in the subject.

CONCLUDING REMARKS

The course introduces informally the basic of chaos theory with the historical developments and highlights the essential characteristics of nonlinear systems. It makes students observe, analyze, and understand unpredictable and uncertain phenomena in the natural or the society via nonlinear thinking.

References

- [1] Smith L.A.: Chaos: a Very Short Introduction. Oxford University Press, Oxford 2007.
- [2] Gleick J.: Chaos: Making a New Science. Viking Press, NY 1987.
- [3] Lorenz E.N.: The Essence of Chaos. University of Washington Press, Seattle 1993.
- [4] Ruelle D.: Chance and Chaos. Princeton University Press, Princeton 1991.
- [5] Stewart I.: Does God Play Dice? The New Mathematics of Chaos. Blackwell Publishing, Oxford 1989.
- [6] Smith P.: Explaining Chaos. Cambridge University Press, Cambridge 1998.
- [7] Lorenz E. N.: Deterministic Nonperiodic Flow. *J. Atmos. Sci.* **20**: 130-141, 1963.
- [8] May R.M.: Biological Populations with Nonoverlapping Generations: Stable Points, Stable Cycles, and Chaos. *Science* **186**: 645-647, 1974.
- [9] Li T.-Y., Yorke J.: Period Three Implies Chaos. *Amer. Math. Monthly* **82**: 985-992.
- [10] Hénon M.: A Two-dimensional Mapping with a Strange Attractor. *Commun. Math. Phys.* **50**: 69-79, 1976.
- [11] May R. M.: Simple Mathematical Models with Very Complicated Dynamics. *Nature* **261**: 459-467, 1976.
- [12] Smale S.: Finding a Horseshoe on the Beaches of Bio. *Math. Intel.* **20**: 39-44, 1998.