ANALOGY STUDY OF ENGINEERING MECHANICS

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Summary Engineering Mechanics plays the role of a bridge or tie between fundamental and specialized knowledge for a four year undergraduate. It is manifested that this subject is difficult for most of the students for they have to deal with numerous concepts and complex formulas. Here we stress that it's convenient and efficient to take a bird's-eye view on the knowledge points if the students grasp the analogy relations in Engineering Mechanics. These relations include the analogies between linear variables and angular variables in dynamics, the analogies between stresses and deformation of all kinds of fundamental deformation styles, the similarity between the large deformation of a beam and the fixed-axis rotation of a pendulum, and the similarity between the buckling of a slender rod and the kinetic stability of a ball.

INTRODUCTION

Engineering Mechanics plays the role of a bridge or tie between fundamental and specialized knowledge for a four year undergraduate. It lays the foundation of nearly all engineering applications, such as mechanical engineering, civil engineering, aerospace engineering, petroleum engineering and ocean engineering. On one hand, it has a wide spectrum of applications, which is different from the more elementary subjects as biology, mathematics, physics and chemistry; on the other hand, although it belongs to "Engineering Science" proposed by Prof. Tsien Hsueshen, it still captures the task of exploring physical laws in nature.

Due to the numerous concepts and complex formulas in Engineering Mechanics, it has been regarded as the most difficult course for a lot of the students. To solve this problem, we proposed a novel study method to master the knowledge points in Engineering Mechanics. It shows that it's convenient and efficient to take a bird's-eye view on this course if the students grasp the analogy relations in Engineering Mechanics. These relations include the analogies between linear variables and angular variables in dynamics, the analogies between stresses and deformation of all kinds of fundamental deformation styles, the similarity between the large deformation of a beam and the fixed-axis rotation of a pendulum, and the similarity between the buckling of a slender rod and the kinetic stability of a ball. The detailed analogy relations are listed as below.

LINEAR & ANGULAR VARIABLES ANALOGY

Mechanics variable	Linear variable	Angular variable	Relation
Displacement	$\boldsymbol{r}=\boldsymbol{r}(t)$ $s=s(t)$	$\varphi=\varphi(t)$	$s=\varphi r$
Velocity	$\boldsymbol{v}=\dot{\boldsymbol{r}}$ $v=\dot{s}$	$\omega=\dot{\varphi}$	$v=\omega r$
Acceleration	$\boldsymbol{a}=\dot{\boldsymbol{v}}=\ddot{\boldsymbol{r}}$ $a_\tau=\dot{v}=\ddot{s}$	$\varepsilon=\dot{\omega}=\ddot{\varphi}$	$a_\tau=\varepsilon r$
Inertial variable	m	J	$J=\sum m_i r_i^2$
Force variable	$\boldsymbol{F}=m\boldsymbol{a}$	$M=J\varepsilon$	$\boldsymbol{M}_o(\boldsymbol{F})=\boldsymbol{r}\times\boldsymbol{F}$ $\boldsymbol{L}_o=\boldsymbol{M}_o(m\boldsymbol{v})=\boldsymbol{r}\times(m\boldsymbol{v})$
Momentum	$\boldsymbol{P}=m\boldsymbol{v}$	$L_0=J\omega$	
Principle of momentum	$\dfrac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t}=\sum\boldsymbol{F}$	$\dfrac{\mathrm{d}\boldsymbol{L}_o}{\mathrm{d}t}=\sum\boldsymbol{M}$	
Kinetic energy	$T=\dfrac{1}{2}mv^2$	$T=\dfrac{1}{2}J\omega^2$	$\mathrm{d}T=\mathrm{d}W$ $T_2-T_1=W$
Work	$W=\int_C F\mathrm{d}s$	$W=\int_\varphi M\mathrm{d}\varphi$	

STRESSES & DEFORMATION ANALOGY

Deformation style	Internal force	Stress	Strength criterion	Deformation	Stiffness	Hooke's law
Tension & Compression	N	$\sigma = \dfrac{N}{A}$	$\sigma_{\max} = \dfrac{N_{\max}}{A} \le [\sigma]$	$\mathrm{d}(\Delta l) = \dfrac{N\mathrm{d}x}{EA}$ $\Delta l = \dfrac{Nl}{EA}$	EA	$\sigma = E\varepsilon$
Shear	Q	$\tau = \dfrac{Q}{A}$	$\tau_{\max} = \dfrac{Q_{\max}}{A} \le [\tau]$	$\mathrm{d}\phi = \alpha \dfrac{Q\mathrm{d}x}{GA}$	GA	$\tau = G\gamma$
Torsion	T	$\tau = \dfrac{T}{I_{\mathrm{p}}}\rho$	$\tau_{\max} = \dfrac{T_{\max}}{W_{\mathrm{p}}} \le [\tau]$	$\mathrm{d}\varphi = \dfrac{T\mathrm{d}x}{GI_{\mathrm{p}}}$ $\varphi = \dfrac{Tl}{GI_{\mathrm{p}}}$	GI_{p}	$\tau = G\gamma$
Bending	M Q	$\sigma = \dfrac{M}{I_{\mathrm{z}}} y$ $\tau = \dfrac{Q}{I_{\mathrm{z}}} \dfrac{S_{\mathrm{z}}^{*}}{b}$	$\sigma_{\max} = \dfrac{M_{\max}}{W_{\mathrm{z}}} \le [\sigma]$ $\tau_{\max} = \dfrac{Q_{\max}}{I_{\mathrm{z}}} \dfrac{S_{\mathrm{z,max}}^{*}}{b} \le [\tau]$	$\mathrm{d}\theta = \dfrac{M\mathrm{d}x}{EI_{\mathrm{z}}}$ $y'' = \dfrac{M}{EI_{\mathrm{z}}}$	EI_{z}	$\sigma = E\varepsilon$
Combined deformation	N+M+T +Q	$\sigma = \sigma^{N} + \sigma^{M}$ $\tau = \tau^{T} + \tau^{M}$	$\sigma_1 - \sigma_3 \le [\sigma]$ $\sigma_{\mathrm{eq}} \le [\sigma]$	$\boldsymbol{u} = \boldsymbol{u}^{N} + \boldsymbol{u}^{M} + \boldsymbol{u}^{T}$	——	$\boldsymbol{\sigma} = \boldsymbol{C} : \boldsymbol{\varepsilon}$

ELASTICA & PENDULUM

The governing equation of a beam enduring the large displacement is the elastica equation:

$$\ddot{\theta} + \alpha^2 \sin\theta = 0 , \qquad (1)$$

and the governing equation of a pendulum is

$$\ddot{\varphi} + \beta^2 \sin\varphi = 0 . \qquad (2)$$

It's clear that the above equations take the same format.

BUCKLING & KINETIC STABILITY

For the Euler rod, if the external load is much smaller than the critical load, it will keep the straight line shape, which is termed as stability. However, when the external load exceeds the critical value, it will lose stability and take the bending shape, which process is called buckling. It's similar to a ball located on a curved substrate. When the ball is spotted at the lowest position, it is stable; otherwise, it won't be in the most stable state.

CONCLUSIONS

In this study, we proposed a novel study method to master the knowledge points in Engineering Mechanics. It shows that it's convenient and efficient to take a bird's-eye view on this course if the students grasp the analogy relations in Engineering Mechanics. It is interesting to make the thick book into only one sheet, and will attract the students' concentration on this course.