DYNAMIC CRUSHING OF HEXAGONAL HONEYCOMBS WITH VARIOUS CELL-WALL ANGLES

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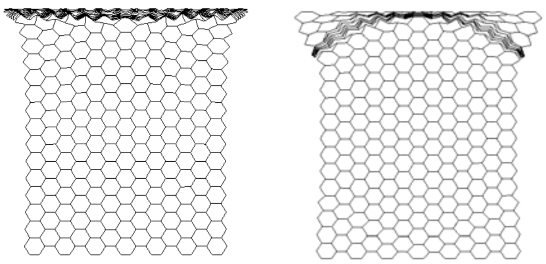
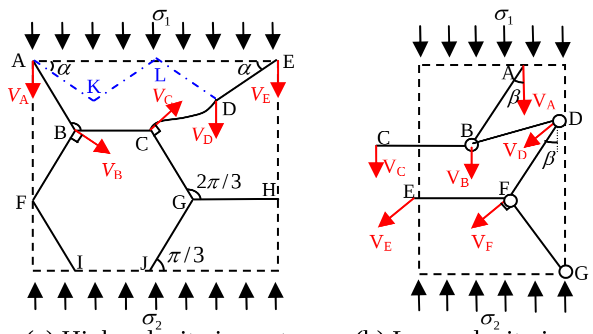
Summary. In-plane dynamic crushing of hexagonal honeycombs with various cell-wall angles is studied by both numerical simulations and analytical models. Honeycombs' deformation modes are classified based on the localization bands and cells' collapse mechanisms. Analytical models are established under both high and low velocity impact respectively to deduce the expressions of honeycombs' mechanical properties, such as the crushing strength, the supporting stress and the critical velocity between the deformation modes.

DEFORMATION MODES

To further understand the dynamic deformation behaviours of the metal hexagonal honeycomb, a finite element model is built using ANSYS-LSDYNA. The hexagonal honeycomb block consists of 21×13 cells, which is placed on a fixed rigid base at the supporting end and in-plane crushed by a rigid plate with a constant crushing velocity V at the loading end. The honeycombs exhibit distinct deformation modes under impact, which are classified into high-velocity mode and low-velocity mode. Under high velocity impact, a localized crushing band is initiated at the loading edge perpendicular to the impact direction and continues to propagate layer by layer to the supporting end. The cell walls are packed closely along the ripple front of the densified region, as shown in Fig. 1(a). In low-velocity deformation mode, cells collapse along the "V" shape localization band layer by layer, forming a stepped front of the densified region, as shown in Fig. 1(b).



(a) High velocity impact (b) Low velocity impact
Fig. 1 Deformation mode



(a) High velocity impact (b) Low velocity impact
Fig. 2 Representative block

CRUSHING STRENGTH

By averaging the stress in the plateau stage of the honeycomb's stress-strain curve, the crushing strength is defined. In view of the periodic characteristics of hexagonal honeycombs both in the geometric structure and in the cells' collapse process, the representative blocks within the honeycomb are employed for the high-velocity and low-velocity deformation mode respectively, as shown in Fig. 2. The movement characteristics of the cell walls involved in the representative blocks are endued according to the numerical results.

Crushing strength under high velocity impact

Under high velocity crushing, the representative block is assumed to have no transverse expansion perpendicular to the impact direction. The stress at the supporting end of the honeycomb is assumed to keep a constant at its limit value, $\frac{2}{3}\sigma_{ys}(\frac{h}{l})^2$, over the collapse period of cells, where σ_{ys} is the yield stress of the base material, h and l are the thickness and the length of the cell wall, respectively. Then based on the repeatable collapse mechanism of the cells' structure and the theorem of linear momentum, the crushing strength of a regular hexagonal honeycomb under high velocity crushing, σ_1^{H}, is obtained as:

$$\sigma_1^{\mathrm{H}} = \frac{2}{3}\sigma_{ys}(\frac{h}{l})^2 + \frac{\rho^*}{1-\frac{8}{3\sqrt{3}}(\frac{h}{l})}V^2 = \frac{2}{3}\sigma_{ys}(\frac{h}{l})^2 + \frac{\rho^*}{1-(\frac{1}{\cos\alpha})(\frac{\rho^*}{\rho_s})}V^2 \quad (1)$$

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where ρ^* and ρ_s are the density of the honeycomb and the base material, respectively, α is the angle of the front of the densified region, as shown in Fig. 2(a). It is obtained from a structural point-of-view, nevertheless is identical with Reid's semi-empirical expression [1] obtained by treating a cellular material as continuum. The prediction of Eq. (1) is compared with numerical simulation results in Ref. [2], showing a very good agreement with each other.

Crushing strength under low velocity impact

Under low velocity crushing, it is assumed that the plastic hinges form only on the vertexes of the cells. Then based on the cells' repeatable collapsing mechanism and the energy balance theory, an analytical expression of the crushing strength of hexagonal honeycombs under low-velocity deformation mode σ_1^{L} is obtained as:

$$\sigma_1^{\mathrm{L}} = \frac{\sigma_{ys}(\frac{h}{l})^2(\pi-2\beta)+3\rho_s(\frac{h}{l})(\frac{V}{\sin\beta})^2}{4(\cos\beta-\frac{h}{2l})(1+\sin\beta)} \quad (\beta\le\frac{\pi}{6}) \quad \text{and} \quad \frac{\sigma_{ys}(\frac{h}{l})^2(\pi-2\beta)+3\rho_s(\frac{h}{l})(\frac{V}{\sin\beta})^2}{4(\cos\beta-\frac{h}{l}\sin\beta)(1+\sin\beta)} \quad (\beta>\frac{\pi}{6}), \tag{2}$$

where β is the cell-wall angle of the honeycomb as marked in Fig. 2(b). Combining Eq. (1) and Eq. (2), the crushing strength of honeycombs is expressed over large variety of crushing velocities.

SUPPORTING STRESS

The honeycomb's stress at the supporting end is related to the protection efficiency when it is used as defence structures. Under low velocity impact, using the theorem of linear momentum in the representative block as shown in Fig. 2(b), and combining Eq. (2), the supporting stress under low-velocity deformation mode σ_2^{L} is obtained as

$$\sigma_2^{\mathrm{L}} = \frac{\sigma_{ys}(\frac{h}{l})^2(\pi-2\beta)-3(\frac{h}{l})\rho_s V^2}{4(\cos\beta-\frac{h}{2l})(1+\sin\beta)} \quad (\beta\le\frac{\pi}{6}) \quad \text{and} \quad \frac{\sigma_{ys}(\frac{h}{l})^2(\pi-2\beta)-3(\frac{h}{l})\rho_s V^2}{4(\cos\beta-\frac{h}{l}\sin\beta)(1+\sin\beta)} \quad (\beta>\frac{\pi}{6}). \tag{3}$$

Both Eqs. (2) and (3) have good agreements with the numerical simulation results. It is shown in Eq. (3) that the supporting stress of the honeycomb increases with the cell-wall angle, but decreases with the crushing velocity in a square law, opposite to the variation of the crushing strength expressed by Eq. (2).

CRITICAL VELOCITY

The supporting stress of the honeycomb on the fixed rigid plate, σ_2^{L}, should always not be less than 0 unless the fixed plate can apply a tensile force on the honeycomb, but the latter is impossible in the present cases. Thus according to $\sigma_2^{\mathrm{L}}\ge 0$, the critical velocity between low-velocity and high-velocity deformation modes is expressed as

$$V_c = \sqrt{\frac{\sigma_{ys}(\pi-2\beta)(\frac{h}{l})}{3\rho_s}}\ , \tag{4}$$

which also gives a limit condition for the application of Eqs. (2) and (3). Assuming a 'shock' type response appear in the cellular material, Tan and Reid [3] proposed a semi-empirical expression of the critical crushing velocity, which depends on the mechanical properties of the cellular material under quasi-static compression. A comparison reveals that Eq. (4) provides predictions almost identical to the formula given in [3].

CONCLUSIONS

Analytical formulas of honeycombs' crushing strength are derived under both high and low velocity impact. An analytical expression of supporting stress under low velocity impact is obtained. The critical velocity between high-velocity and low-velocity deformation modes is also deduced analytically.

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