PROBABILISTIC HOMOGENIZATION OF SOLID FOAMS WITH APPLICATION TO SANDWICH STRUCTURES

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Summary The present study is concerned with a numerical analysis of the scatter in the effective material response of solid foams deriving from the microstructural disorder of the material. The effective stress and strain components are determined numerically by means of an volume averaging procedure. In a probabilistic enhancement, the approach is applied to small-scale testing volume elements in order to obtain the stochastic information regarding the local scatter in the effective material properties. As a case study, a single edge clamped sandwich beam with foam core is considered. The material parameters of the foam core are defined as a random field whose properties are determined by the probabilistic homogenization procedure.

INTRODUCTION

Solid foams are important materials in lightweight construction where materials are required which combine a rather low specific weight with a reasonable macroscopic stiffness and strength. On the other hand, foamed materials feature a random disordered microstructure leading to a distinct scatter in the macroscopic material properties (e.g. Fazekas et al. [2]). Aim of the present study is the definition of a probabilistic homogenization procedure for a numerical prediction of the mean values and the scatter in the macroscopic material properties, based on the knowledge of the microstructural geometry and topology and the corresponding local variations and uncertainties.

PROBABILISTIC HOMOGENIZATION PROCEDURE

General concept

The numerical prediction of the effective properties of the solid foam is based on the concept of the representative volume element. Hence, for analysis of a body Ω according to Figure 1, a representative volume element Ω^{RVE} for the microstructure is considered. This volume element is replaced by a similar volume element $\Omega^{\mathrm{RVE}*}$ consisting of the "effective" medium. In an analysis of the response of the two volume elements under a number of independent reference deformations, the material behaviour of $\Omega^{\mathrm{RVE}*}$ is determined such that the behaviour of both volume elements is similar. The mesoscopic equivalence of the two volume elements is defined by the condition that the volume averages

$$\bar{\boldsymbol{\varepsilon}} = \frac{1}{\Omega^{\mathrm{RVE}}} \int\limits_{\Omega^{\mathrm{RVE}}} \boldsymbol{\varepsilon}\,\mathrm{d}V = \frac{1}{\Omega^{\mathrm{RVE}*}} \int\limits_{\Omega^{\mathrm{RVE}*}} \boldsymbol{\varepsilon}^*\,\mathrm{d}V \quad , \quad \bar{\boldsymbol{\sigma}} = \frac{1}{\Omega^{\mathrm{RVE}}} \int\limits_{\Omega^{\mathrm{RVE}}} \boldsymbol{\sigma}\,\mathrm{d}V = \frac{1}{\Omega^{\mathrm{RVE}*}} \int\limits_{\Omega^{\mathrm{RVE}*}} \boldsymbol{\sigma}^*\,\mathrm{d}V \qquad (1)$$

of the strains and stresses in both volume elements have to be equal for all reference deformations. The effective material properties defining the relation between the effective strains $\bar{\boldsymbol{\varepsilon}}$ and stresses $\bar{\boldsymbol{\sigma}}$ are then used for the analysis of the body Ω.

Computational foam model

The microstructure of the representative volume elements is determined randomly by means of a Voronoï process in Laguerre geometry (Fan et al. [1]). In this context, a number of n nuclei i is positioned randomly within Ω^{RVE}. All nuclei are supplied with surrounding non-intersecting spheres of radius r_i. Subsequently, the cell walls are defined as the sets of all spatial points with equal distances to the two nearest sphere surfaces (Figure 2). For a finite element analysis of the model foam, the cell walls are then meshed with standard displacement based triangular shell elements.

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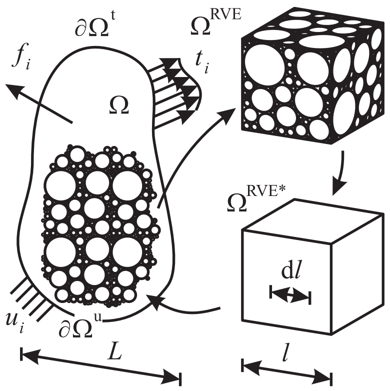


Figure 1. RVE concept.

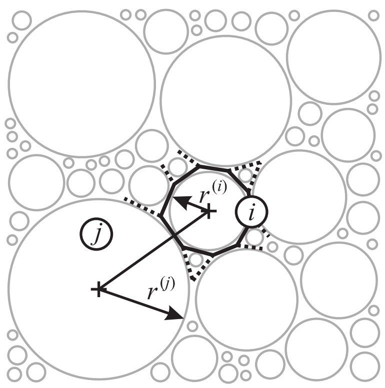


Figure 2. Voronoï-Laguerre process.

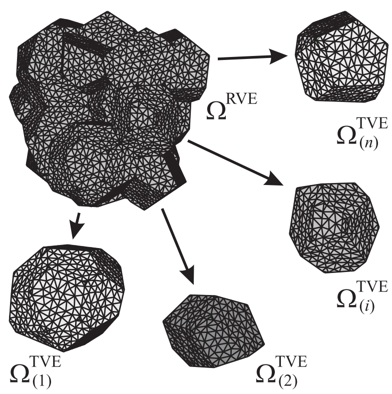
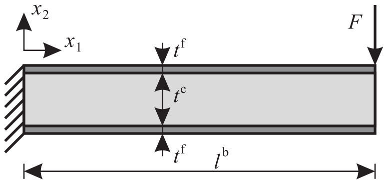


Figure 3. Testing volume elements.



geometry & load	face sheet material	core material
$l^{\mathrm{b}} = 500$ mm	$E^{\mathrm{c}} = 70$ GPa	$E^{\mathrm{base}} = 70$ GPa
$t^{\mathrm{f}} = 1$ mm	$\nu^{\mathrm{c}} = 0.3$	$\nu^{\mathrm{base}} = 0.3$
$t^{\mathrm{c}} = 25$ mm		$\overline{\rho}^{\mathrm{c}} = 0.08$
$F = 0.1$ kN		$\overline{V}^{\mathrm{c}} = 3.9\ \mathrm{mm}^3$

Figure 4. Structural example.

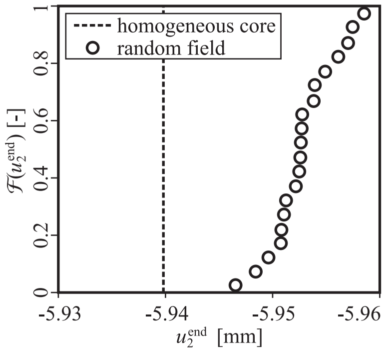


Figure 5. End deflection.

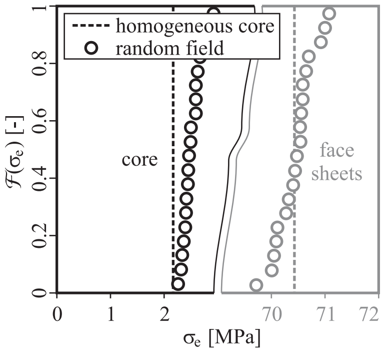


Figure 6. Maximum stress.

Local effective properties

The computational foam models according to Figure 2 in general have to be rather large in order to be statistically representative (i.e. $L \gg l \gg \mathrm{d}l$). If this condition cannot be satisfied, as in the case of sandwich cores with a thickness in the range of $L = t^{\mathrm{c}} \approx 20\,\mathrm{mm}, \ldots 50\,\mathrm{mm}$ made from large cell foams with cell diameters in the range of $1\,\mathrm{mm}$, the computed effective properties are subject to scatter. In order to assess the uncertainty, the homogenization procedure is applied to "testing volume elements" Ω^{TVE} defined as subsets of a large-scale, statistically representative volume element Ω^{RVE}. Since the entire set of testing volume elements $\Omega^{\mathrm{TVE}}_{(i)}$ defines the representative volume element, the set of material parameters based on the individual TVE analyses is assumed to define the total range of uncertainty for the material parameters on the respective length scale. In the present study, the individual cells of the foam are considered as the smallest feasible testing volume elements (Figure 3). Their effective properties are determined by applying a number of individual reference deformation states to the entire representative volume elements and application of the homogenization equations (1) to the individual testing volume elements. The results are evaluated by stochastic methods.

CASE STUDY

As a structural example, a single edge clamped sandwich beam according to Figure 4 with aluminum face sheets and a closed cell aluminum foam core subjected to a point force on its free edge is consicdered. The effective stiffnesses of the core and their uncertainties are determined by means of the probabilistic homogenization procedure described in the previous section. Based on these results together with the autocorrelation of the material properties at neighboring positions of the effective continuum, the material properties for the foam core in the structural example are defined as a random field. In the case study, a total of 20 numerical experiments for generation of the random field and a subsequent finite element analyses of the example problem is performed.

RESULTS AND DISCUSSION

In Figures 5 and 6, the results for the accumulated probability distributions $\mathcal{F}$ for the deflection u_2^{end} of the free end of the beam and the maximum value of the v. Mises equivalent stress σ_{e} are presented together with the corresponding results based on the assumption of a homogeneous core stiffness instead of the random fields. For the end deflection, the scatter in the microstructural disorder of the core has only insignificant effects with a scatter band width for the end deflection of $\Delta u_2^{\mathrm{end}} \approx 0.015\,\mathrm{mm}$. Nevertheless, compared to the analysis with a homogeneous core, the use of the random field results in a slightly weaker structural response with an increase of the median value of the end deflection by about 1%. For the maximum values of the equivalent stress σ_{e} used in strength analyses for the core and the face sheets in Figure 6, more distinct effects of the microstructural disorder are observed. For the core, a the random field formulation results in an increase of the median equivalent stress by about 13% with a scatter band width of 18% of the corresponding median value. For the face sheets, the use of the random field for the core stiffness results in an uncertainty in the equivalent stress by about $\pm 0.7\,\mathrm{MPa}$. The results reveal that the microstructural disorder might have distinct effects especially in the strength analysis, where a local stress maximum rather than an integral average forms the relevant parameter. Care has to be taken in simplified approaches assuming a homogeneous stiffness field instead of the random field since this type of analysis does not only neglects the scatter but might also underestimate the median value of the relevant equivalent stress.

References

[1] Fan Z., Wu Y., Zhao X. Lu, Y.: Simulation of polychrystalline structure with Voronoi diagram in Laguerre geometry based on random closed packing of spheres. *Comput Mat Sci* **29**:301-308, 2004.

[2] Fazekas A., Dendievel R., Salvo L., Bréchet Y.: Effect of microstructural topology upon the stiffness and strength of 2D cellular structures. *Int J Mech Sci* **44**:2047-2066, 2002.