SIZE-DEPENDENT AND TUNABLE ELASTIC PROPERTIES OF HIERARCHICAL HONEYCOMBS AND OPEN-CELLED FOAMS

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Summary Based on the perfect regular self-similar hexagonal and tetrakaidecahedon structure models and the Timoshenko beam theory, analytical results for the elastic properties of hierarchical honeycombs and open-celled foam are obtained. It is found that for both hierarchical honeycombs and open-celled foams with the first order cell wall thickness or strut diameter at nanometer scale, not only their elastic properties are size-dependent, but also can be controlled to vary over a large range.

HIERARCHICAL HONEYCOMBS

1. Size-dependent and Tunable Elastic Properties of the 1st Order Nano-sized Honeycombs

The dimensionless Young's modulus and Poisson ratio of the 1st order regular honeycombs have been obtained as [1, 2]

$$\overline{E_1} = \frac{E_1}{1.5E_0\rho_1^3/(1-v_0^2)} = \frac{1-v_0^2}{1.5E_0\rho_1^3}\cdot\frac{16\sqrt{3}D_bD_sD_c}{D_sD_cL^3W+12D_bD_cLW+36D_sD_bLW} \tag{1}$$

Where, E_0 and v_0 are the Young's modulus and Poisson ratio of the solid material; $\rho_1 = 2h_0/(\sqrt{3}L_0)$ is the honeycomb relative density, L_0, W_0 and h_0 are the cell wall length, width and thickness, respectively, when the initial surface stress is absent; $h = h_0(1+\varepsilon_0^h)$, $L = L_0(1+\varepsilon_0^L)$, and $W = W_0(1+\varepsilon_0^W)$; $\varepsilon_0^h = -2v_0\sigma_0^L/E_0 = 4v_0\tau_0/(E_0h)$ and $\varepsilon_0^L = \varepsilon_0^W = \sigma_0^L(1-v_0)/E_0 = -2\tau_0(1-v_0)/(E_0h)$; and τ_0 is the initial surface stress. D_b, D_s and D_c are the size-dependent bending stiffness, transverse shear stiffness and the axial tensile/compressive stiffness of the cell walls, and given as [2, 3]

$$D_b = \frac{Wh^3}{12}[1+6\frac{l_n}{h}+\frac{v_0(1+v_0)}{1-v_0}\varepsilon_0^L] \tag{2}$$

$$D_s = \frac{G_0Wh}{1.2}\cdot\frac{[1+\frac{6l_n}{h}+\varepsilon_0^Lv_0\frac{(1+v_0)}{1-v_0}]^2}{1+\frac{10l_n}{h}+30(\frac{l_n}{h})^2} \tag{3}$$

$$D_c = E_0Wh(1+2l_n/h) \tag{4}$$

Where $l_n = S/E_0$ is the intrinsic length of the material at the nanoscale and S is the surface elasticity modulus. As the initial surface stress can be controlled to vary by adjusting the amplitude of an applied electric potential, the elastic properties of the 1st order nano-honeycomb are not only size-dependent, but also tunable over a large range. Figures 1a and 1b show the size-dependent and tunable dimensionless Young's modulus of the 1st order regular nano-sized hexagonal honeycombs. Zhu [3] has studied the size-dependent elastic properties of nano-sized honeycomb, but not yet considered the effects of the dimension change of the cell walls (due to the change of the initial surface stress) on the results.

2. Size-dependent and Tunable Elastic Properties of the Self-similar Hierarchical Honeycombs

For the self-similar hierarchical hexagonal honeycombs, the relative density of the *n*th order hierarchical honeycomb can be obtained as $\rho_n = (\rho_1)^n$. For different order of honeycombs, their out-of-plane Poisson ratio is 0. The in-plane Poisson ratio of the *n*th order hierarchical honeycomb is v_1, the same as that of the 1st order honeycomb. The dimensionless Young's modulus of the *n*th order hierarchical honeycomb can be obtained as

$$\overline{E_n} = \frac{E_n}{1.5E_0\rho_1^3/(1-v_0^2)} = [\frac{1.5\rho_1^3}{1+1.8(2.25+v_1)\rho_1^2}]^{n-1}\overline{E_1} \tag{5}$$

Where, $\overline{E_1}$ is given in Eq. 1 and figures 1a and 1b. Thus, the elastic properties of the *n*th order hierarchical honeycomb with the 1st order cell wall thickness at the nanoscale are also size-dependent and tunable.

HIERARCHICAL OPEN-CELLED FOAMS

3. Size-dependent and Tunable Elastic Properties of the 1st Order Open-celled Foams

For simplicity, the cell struts are assumed to have a circular cross-section. The dimensionless Young's modulus and Poisson ratio of the 1st order nano-sized regular tetrakaidecahedon open-celled foams can be obtained as

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$$\overline{E}_1 = \frac{E_1}{0.6E_0\rho_1^2} = \frac{1}{0.6E_0\rho_1^2}\cdot\frac{6\sqrt{2}\cdot D_bD_SD_C}{D_SD_CL^4+14.4D_bD_CL^2+12D_bD_SL^2} \quad (6)$$

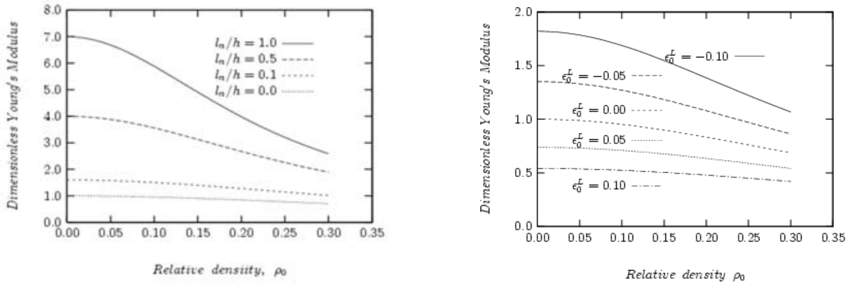
Where, $\rho_1 = 3\pi d_0^2/(8\sqrt{2}L_0^2)$ is the foam relative density, L_0 and d_0 are the cell strut length and cross-sectional diameter when the initial surface stress is absent. When the initial stress is present, $d = d_0(1+\varepsilon_0^r)$ and $L = L_0(1+\varepsilon_0^L)$, $\varepsilon_0^r = -2\tau_0(1-3\nu_0)/(E_0d) = \sigma_0^L(1-3\nu_0)/(2E_0)$, $\varepsilon_0^L = -4\tau_0(1-\nu_0)/(E_0d) = \sigma_0^L(1-\nu_0)/E_0$, and $\sigma_0^L = -4\tau_0/d$. E_0, τ_0 and l_n are the same as those for nano-honeycombs. From [4], $D_b = (\pi d^4E_0/64)[1+8l_n/d+\nu_0\varepsilon_0^L(1-3\nu_0)/(1-\nu_0)]$. The stretching stiffness of the struts with a circular cross-section is $D_C = (\pi d^2/4)(1+4l_n/d)$. As there is no closed form result for the transverse shear stiffness of core-shell structures, the shear stiffness of the nano-struts is assumed to be $D_S = (9\pi d^2/40)(1+2l_n/d)$. The size-dependent and tunable dimensionless Young's modulus of the 1st order nano-sized regular open-celled foams are shown in figures 2a and 2b.

4. Size-dependent and Tunable Elastic Properties of the Self-similar Hierarchical Open-celled Foams

For the self-similar hierarchical open-celled foams, the relative density of the *n*th order foam can be obtained as $\rho_n = (\rho_1)^n$ and the Poisson ratio of the *n*th order hierarchical open-celled foam is ν_1, the same as that of the 1st order open-celled foam. The dimensionless Young's modulus of the *n*th order hierarchical honeycomb can be obtained as

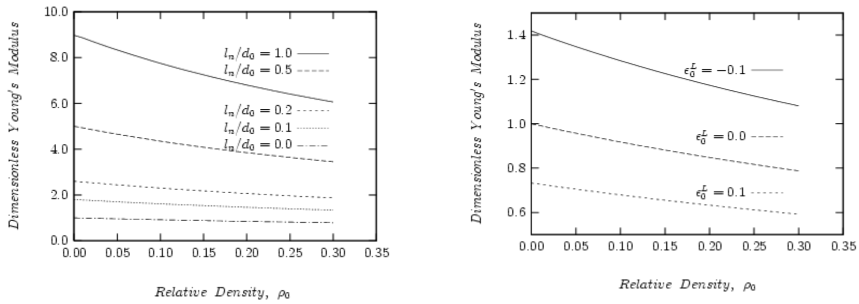
$$\overline{E_n} = \frac{E_n}{0.6E_0\rho_1^2} = [\frac{0.6\rho_1^2}{1+(1.62(1.556+\nu_1)\rho_1}]^{n-1}\overline{E_1} \quad (7)$$

Where, $\overline{E_1}$ is given in Eq. (6) and figures 2a and 2b.



(a) when the effect of initial surface stress is absent. (b) when the effect of surface elasticity modulus is absent.

Fig. 1. Size-dependent and tunable elastic properties of the 1st order nano-sized honeycombs.



(a) when the effect of initial surface stress is absent. (b) when the effect of surface elasticity modulus is absent.

Fig. 2. Size-dependent and tunable elastic properties of the 1st order nano-sized open-celled foams.

CONCLUSIONS

If the cell wall thickness or diameter of the 1st order honeycomb or open-celled foam is at micrometer scale, the elastic properties the hierarchical honeycomb or open-celled foam are size-dependent; if the thickness or diameter is at nanometer scale, the elastic properties of the hierarchical honeycomb or open-celled foam are not only size-dependent, but also tunable.

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