THERMAL STRESSES AROUND A CYLINDRICAL HOLE IN A TRANSVERSELY ISOTROPIC POROELASTIC MEDIUM CONSIDERING LOCAL THERMAL NON-EQUILIBRIUM

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Summary This work presents a local thermal non-equilibrium (LTNE) poroelasticity model for transversely isotropic porous media and investigates the thermally induced pore pressure and thermal stresses around a cylindrical hole in an infinite porous medium subjected to convective cooling on the boundary of the cavity. The LTNE thermo-poroelasticity equations are solved using Laplace transform and numerical examples are presented to examine the effects of LTNE and material anisotropy on the pore pressure and thermal stresses around the cavity. The results show that the LTNE effects become more pronounced when the convective heat transfer boundary conditions are employed. The thermally induced pore pressure and the magnitude of thermal stresses are significantly higher than the corresponding values in the classical poroelasticity for moderate Biot numbers. The results also show that the pore pressure and the magnitude of thermal radial stress decrease with decreasing elastic modulus in the transverse direction.

INTRODUCTION

In the classical thermo-poroelasticity of fluid saturated porous media, the conditions of local thermal equilibrium are assumed, i.e., there is no heat transfer between the solid and fluid phases at the microscopic level and the temperatures of the solid and fluid at a macroscopic point remain the same. The assumption of local thermal equilibrium remains adequate for many engineering and geophysical applications when the Sparrow number is very large [1] which corresponds to large interstitial heat transfer coefficients between the solid and fluid phases and high pore surface area to pore volume ratios. For applications with small Sparrow numbers, local thermal non-equilibrium (LTNE) becomes significant, which promotes heat transfer between the solid and fluid and induces additional pore pressure and thermal stresses in the porous medium. In these circumstances, LTNE heat transfer/poroelasticity theories should be used to precisely evaluate the pore pressure and thermal stresses in the porous media. Effects of LTNE on thermal stresses have been addressed in a few studies (e.g., [2] and [3]). However, the anisotropy of the porous media has not been taken into account. It is well known that geological formations, especially shale formations are anisotropic materials. In this paper, we present an LTNE poroelasticity model for transversely isotropic porous media and investigates the effects of LTNE and material anisotropy on the pore pressure and thermal stress distributions around a cylindrical hole in an infinite porous medium subjected to convective cooling at the boundary of the cavity.

AN LTNE POROELASTICITY MODEL FOR TRANSVERSELY ISOTROPIC POROUS MEDIA

Consider a transversely isotropic, fluid saturated porous medium undergoing local thermal non-equilibrium (LTNE). Under the LTNE condition, the temperatures of the solid and fluid phases are governed by the following heat conduction equations

$$(1-\phi_0)\rho_s c_s(\partial\theta_s/\partial t) = (1-\phi_0)\left[k_{Ts1}\left(\partial^2\theta_s/\partial x^2 + \partial^2\theta_s/\partial y^2\right) + k_{Ts3}\left(\partial^2\theta_s/\partial z^2\right)\right] + h_{\text{int}}(\theta_f - \theta_s) \tag{1a}$$

$$\phi_0\rho_f c_f \frac{\partial\theta_f}{\partial t} = \phi_0 k_{Tf}\nabla^2\theta_f - h_{\text{int}}(\theta_f - \theta_s) \tag{1b}$$

where θ_s and θ_f are the solid and fluid temperature variations, respectively, t time, ϕ_0 the reference porosity, k_{Tf} the thermal conductivity of the fluid, k_{Ts1} and k_{Ts3} the thermal conductivities of the solid in the isotropic plane (x-y plane) and the transverse direction, respectively, ρ_s and ρ_f the densities for the solid and fluid, respectively, c_s and c_f the specific heat for the solid and fluid, respectively, h_{int} the solid-fluid interface heat transfer coefficient, and ∇^2 the Laplacian operator. The heat transfer by convection is neglected in this study. The LTNE heat equations (1a) and (1b) reduce to those for the isotropic media [4] when $k_{Ts1} = k_{Ts3}$.

The LTNE constitutive equations may be formulated using the weighted average of the solid and fluid temperatures as follows

$$\begin{aligned}
\sigma_{xx} &= M_{11}\varepsilon_{xx} + M_{12}\varepsilon_{yy} + M_{13}\varepsilon_{zz} - \alpha_1 p - \beta_{s1}\theta_{ave} \\
\sigma_{yy} &= M_{12}\varepsilon_{xx} + M_{11}\varepsilon_{yy} + M_{13}\varepsilon_{zz} - \alpha_1 p - \beta_{s1}\theta_{ave} \\
\sigma_{zz} &= M_{13}\varepsilon_{xx} + M_{13}\varepsilon_{yy} + M_{33}\varepsilon_{zz} - \alpha_3 p - \beta_{s3}\theta_{ave} \\
\tau_{xy} &= 2G_{12}\varepsilon_{xy}, \qquad \tau_{yz} = 2G_{13}\varepsilon_{yz}, \qquad \tau_{xz} = 2G_{13}\varepsilon_{xz}
\end{aligned} \tag{2}$$

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$$p = M\varsigma - M\alpha_1(\varepsilon_{xx} + \varepsilon_{yy}) - M\alpha_3\varepsilon_{zz} + M\beta_{sf}\theta_{ave} \tag{3}$$

where θ_{ave} is the weighted average temperature for the porous medium given by

$$\theta_{ave} = (1-\phi_0)\theta_s + \phi_0\theta_f \tag{4}$$

σ_{ij} denotes stresses ($i, j = x, y, z$), ε_{ij} strains, p the pore pressure, ζ the fluid content variation, M Biot's modulus, α_1 and α_3 Biot's effective stress coefficients in the isotropic plane and the transverse direction, respectively, G_{12} and G_{13} the shear moduli in the isotropic plane (x-y plane) and the x-z plane, respectively, M_{11}, M_{12}, M_{13} and M_{33} the drained elastic constants which can be found in Cheng [5], and β_{s1} and β_{s3} are related to M_{ij}, α_{s1} and α_{s3} (the linear thermal expansion coefficients of the solid in the isotropic plane and transverse direction, respectively), and α_f (the volumetric thermal expansion coefficient of the fluid) given in Abousleiman and Ekbote [6].

A CYLINDRICAL HOLE IN AN INFINITE POROUS MEDIUM

Consider a cylindrical hole of radius a in an infinite transversely isotropic porous medium with the hole in the transverse (z) direction. The boundary of the hole is subjected to a sudden pressure applied by a fluid whose temperature θ_a is different from that of the medium (assumed as zero without loss of generality). This is a fundamental problem for drilling applications in petroleum engineering. The displacement field of this axisymmetrical problem is irrotational. Hence, the governing equations of pore pressure and radial displacement are uncoupled and can be derived as follows

$$\partial p/\partial t = c\nabla^2 p + c'\partial\theta_{ave}/\partial t, \qquad M_{11}(\partial u/\partial r + u/r) = \alpha_1 p + \beta_{s1}\theta_{ave} \tag{5}$$

where
$$c = (k_1/\mu)MM_{11}/(M_{11} + \alpha_1^2 M), \qquad c' = c[\beta_{sf} - \alpha_1\beta_{s1}/M_{11}]/(k_1/\mu) \tag{6}$$

u is the displacement in the radial direction, r ($\geq a$) the radial coordinate, k_1 the intrinsic permeability in the isotropic x-y plane, and μ the fluid viscosity. The temperatures, thermally induced pore pressure and stresses for a sandstone formation are obtained using the Laplace transform method. Here we assume a convective heat transfer condition for the solid phase and a prescribed temperature condition for the fluid phase at the hole boundary ($r = a$). The material and transport properties generally follow those in [3] with the assumption that $\nu_{13}/\nu_{12} = 1.0$ and $\alpha_{s3}/\alpha_{s1} = 1.0$. The modulus ratio E_3/E_1 is varied from 0.5 to 2.0 following Ref. [6], where E_1 and E_3 are the drained moduli in the isotropic plane (x-y) and the transverse direction (z), respectively. Below we present the results for a nondimensional time of $\kappa t/a^2 = 0.1$ and a Biot number of $Bi = 1.0$.

Fig. 1 shows the thermally induced, LTNE (solid lines) and classical (dashed lines) pore pressures normalized by $c'\theta_a \times 10^{-5}$ along the radial direction for various modulus ratios E_3/E_1. The thermal pore pressure first increases with increasing r/a, reaches a peak value, and then decreases with further increases in the distance from the hole boundary. The LTNE peak pore pressure is significantly higher than that of the classical theory. Moreover, the pore pressure decreases with a decrease in the modulus ratio E_3/E_1. Fig. 2 shows the thermally induced LTNE (solid lines) and classical (dashed lines) radial stresses normalized by $c'\theta_a \times 10^{-2}$ along the radial direction. The magnitude of the radial stress initially increases with increasing r/a, reaches a peak value, and then decreases with further increasing distance from the hole. The magnitude of the LTNE peak radial stress is higher than that of the classical solution. Also, the magnitude of the radial stress decreases with a decrease in the modulus ratio E_3/E_1.

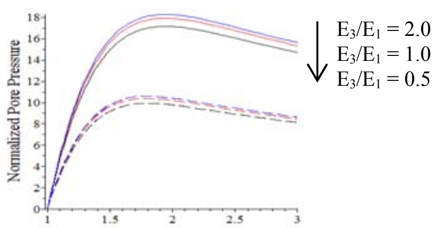

Fig.1 Normalized thermal pore pressures along the radial direction around a cylindrical hole

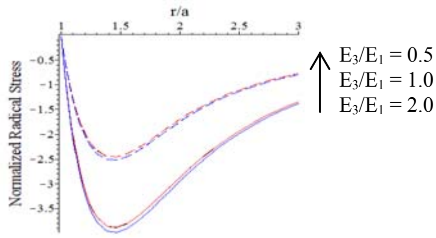

Fig. 2 Normalized thermal radial stresses along the radial direction around a cylindrical hole

References

[1] Minkowycz W. J., Haji-Sheikh A., Vafai K.: On the departure from local thermal equilibrium in porous media due to a rapidly changing heat source: the Sparrow number. *Int. J. Heat Mass Transfer* **42**: 3373-3385, 1999.

[2] He L.-W., Jin Z.-H.: A local thermal nonequilibrium poroelastic theory for fluid saturated porous media. *J. Therm. Stresses* **33**: 799-813, 2010.

[3] He L.-W., Jin Z.-H., Zhang Y.: Convective cooling/heating induced thermal stresses in a fluid saturated porous medium undergoing local thermal nonequilibrium. *Int. J. Solids Struct.*, 2012, in press.

[4] Nield D. A., Bejan, A.: Convection in Porous Media, 3rd edition, Springer, Berlin, 2006.

[5] Cheng A. H.-D.: On the generalized plane strain poroelasticity. *Int. J. Rock Mech. Min. Sci*. **35:** 183-193, 1998.

[6] Abousleiman Y., Ekbote S.: Solutions for the inclined borehole in a porothermoelastic transversely isotropic medium. *J. Appl. Mech*. **72**: 102-114, 2005.