SINGULARITY OF THE ENERGY PROPAGATION IN ANISOTROPIC FLUID-FILLED POROUS MATERIALS

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Summary Based on Biot's theory, the energy propagation singularity in open-cell anisotropic porous solids is studied with the aim to provide a reference in the energy design and application of porous materials. By using the perturbed eigenvalues that arises when the nearly pure modes are propagated, the evolution conditions on the elastic parameters of skeleton materials are established for the existence of various systems of folds on the wave front. The critical conditions for the pattern transformation in the parameter space are given. The results show that the situation in the pure solids is a degenerated case of the present discussion.

INTRODUCTION

Unlike the isotropic situation, anisotropy would cause nonhomogeneous wave propagation in the materials. By now, some researches about the wave singular propagation in porous materials have been carried out [1]. However, the critical transformation conditions for the existence of various fold systems have not been established, especially for slow wave mode, which represents a relative motion between the fluid and the solid frame, and plays an important role in the description of the fluid-solid interaction effects in porous materials. In order to establish the relation between the focusing patterns and material parameters, the energy singular propagation in fluid-filled porous materials is studied in this paper with the purpose to provide some guidance in the experiment data processing, and also in the design and further application of porous materials.

METHODOLOGY

For a general anisotropic fluid-filled porous media, the equations of motion and the constitutive relation in Lagrange coordinate system in the absence of body forces may be expressed as [2]

$$\sigma_{ij,j} = \rho \ddot{u}_i + \rho_f \ddot{w}_i \,, \quad -p_{,i} = \rho_f \ddot{u}_i + m_{ij} \ddot{w}_j + r_{ij} \dot{w}_j \,, \tag{1}$$

where σ_{ij} are the symmetry stress of the solid skeleton, p the fluid pressure. ξ=-div**w** is the variation of the fluid content in which $w_i=\phi(U_i-u_i)$ are the displacements of the fluid relative to the solid frame with ϕ the porosity, u_i and U_i the components of the average displacements of the solid skeleton and fluid phases, respectively. $\rho=(1-\phi)\rho_s+\phi\rho_f$ denotes the bulk-fluid mixture density with ρ_s and ρ_f the densities of the solid grain and the pore fluid, respectively. Repeated indices imply summation. The comma stands for the partial derivative with respect to the space variable x_i and the upper dot the partial derivative to the time t. m_{ij} and r_{ij} are mass and damping symmetric tensors [3]. Otherwise, without special declaration, i, j=1, 2, 3 are corresponding to x, y, z, respectively.
Considering the harmonic solutions of Eqs. (1), that is,

$$[u_i, w_i] = [A_i, B_i] \exp \mathrm{i}\kappa(n_j x_j - vt) \,, \tag{2}$$

we have

$$\begin{bmatrix} \rho v^2 \delta_{ik} - A_{ijkl} n_l n_j & \rho_f v^2 \delta_{ik} - M_{kl} n_l n_i \\ \rho_f v^2 \delta_{ik} - M_{kl} n_l n_i & \overline{m}_{ik} v^2 - M n_i n_k \end{bmatrix} \cdot \begin{bmatrix} A_i \\ B_i \end{bmatrix} = 0, \tag{3}$$

where A_i and B_i are the amplitudes of the displacement components, n_j is the directional cosine, $\kappa=\omega/v$ the wave number with v the phase velocity, $\overline{m}_{ik} = m_{ik} - ir_{ik} / \omega$. In order to obtain the nontrivial solution of Eq. (3), the determinant of the coefficient matrix must be zero, which yields a bi-quadratic characteristic equation for the phase velocities

$$D^V = \begin{vmatrix} \rho v^2 \delta_{ik} - A_{ijkl} n_l n_j & \rho_f v^2 \delta_{ik} - M_{kl} n_l n_i \\ \rho_f v^2 \delta_{ik} - M_{kl} n_l n_i & \overline{m}_{ik} v^2 - M n_i n_k \end{vmatrix} = 0 \,. \tag{4}$$

Its four positive eigenvalues v_l (l=1,2,3,4) correspond to the phase velocities of the fast wave, the slow wave and the two quasi-transverse waves, which are marked as FL, SL, QST, and QFT, respectively.
Let us make an assumption that the propagation direction $\mathbf{n}_1$ is a pure mode propagation direction in the material. If a small error has been made between the real propagation direction $\mathbf{n}_2$ and the pure mode direction $\mathbf{n}_1$, the nearly pure mode propagation problem arises. The perturbed eigenvalues of the system with the wave propagation near the direction $\mathbf{n}_1$ should be obtained. In order to solve this problem, three coordinate systems are established. One is the

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material coordinate $\{\zeta_i\}$, which is given in the bulk material coordinate. The second one is the reference coordinate $\{y_i\}$, which are chosen by the conditions that y_3 be the pure mode axis. The third one is the coordinate $\{\psi_i\}$, referred as "misoriented coordinate", which is established by making a small error θ between the ψ_3 axis and the pure mode axis y_3 [4]. The eigenvalues for the wave propagation along the direction **n** has been given in the material coordinate $\{\zeta_i\}$, in Eq. (4). Then, with the help of the reference coordinate $\{y_i\}$, the perturbed eigenvalues could be solved by two coordinate transformations.

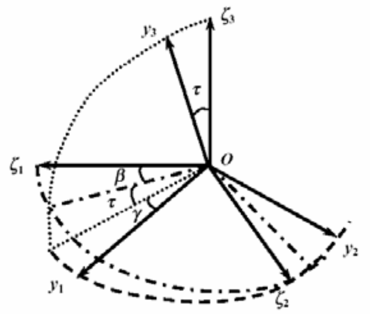

Fig. 1. The diagrammatic sketch for the coordinate transformation the coordinate $\{\zeta_i\}$ to the coordinate $\{y_i\}$.

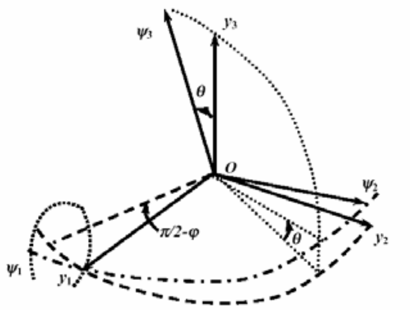

Fig. 2. The diagrammatic sketch for the coordinate transformation from from the coordinate $\{y_i\}$ to the coordinate $\{\psi_i\}$.

For the slowness surface of the fluid-filled porous material, the sections of positive and negative curvatures are separated by parabolic lines with zero Gaussian curvature. The energy flux along the wave front vector, which is normal to the slowness surface, is inversely proportional to the curvature at that point, that is, the parabolic lines on the slowness surface just mathematically correspond to the caustics on the wave fronts [5]. Since the sign changes of the Gaussian curvature indicates the existence of the new fold system, the critical conditions for the focusing pattern transformation are given by

$$\partial\theta_g / \partial\theta = 0, \quad \partial\varphi_g / \partial\varphi = 0\,, \tag{5}$$

where (θ_g, φ_g) the direction of the wave front vector. The critical conditions are then determined, and represented by the lines in Fig. 3, which are marked as line A to line K subsequently. These lines signal the changes of the sign of the Gaussian curvature and the appearance of a new feature. Due to the limitation o the paper, the explicit form for the transforming critical conditions are not given any more.

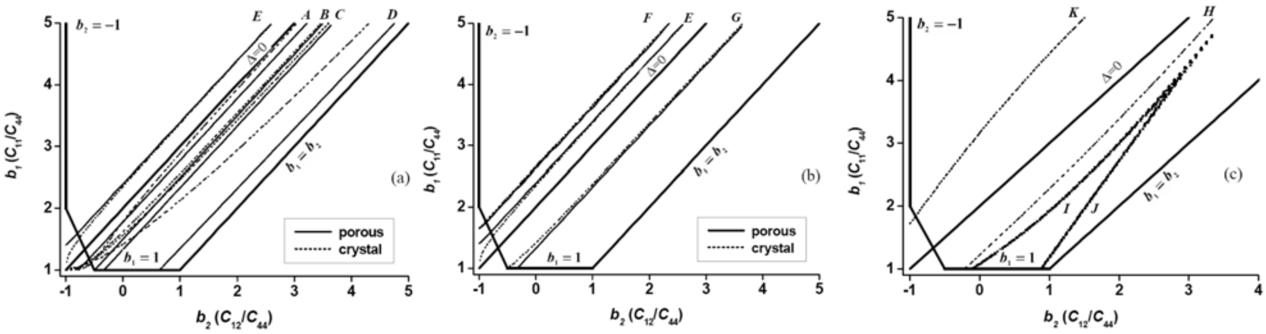

Fig. 3. Conditions on the elastic constants that lead to various singular features in the sheets of the slowness surface and wave front: a) for QST; b) for QFT; c) for SL.

CONCLUSIONS

The critical pattern transformation conditions, as well as the corresponding topology evolution of the energy focusing on the wave fronts, are systematically investigated for fluid-filled porous materials. The critical transformation conditions for the fluid-filled porous materials with cubic symmetry are established, which provides direct information about the possible focusing pattern for a certain fluid-filled porous material without having recourse to the detailed calculating. The emphasis is placed on the focusing occurred on the slow wave front, which is particular for the fluid-filled porous materials. The results will be helpful in the energy design of anisotropic fluid-filled porous materials.

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