A MICROMECHANICAL STUDY ON THE SHEAR STRENGTH IN GRANULAR MATERIALS: THE ROLE OF PARTICLE SHAPE

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Summary This paper presents a micromechanical investigation on the behaviour of granular materials under confined shear. A three-dimensional Discrete Element Method is employed for the study. While spherical particles are used in the study, proper rolling resistance is introduced in the DEM to consider the shape effect of real granular particles. Emphasis is placed on the impact of fabric anisotropy on the characteristics of shear strength and critical state in granular media. It is found that a theoretical expression for the shear strength of a granular assembly, in terms of the contributions from different sources of anisotropy, still correlates very well with the statistical results from DEM when rolling resistance is considered. Higher rolling resistance generally leads to stronger material response.

INTRODUCTION

The collective behaviour of granular media on the continuum level is known to be closely related to the underlying microstructures formed at grain scale. Discrete Element Method (DEM) has become a particularly popular tool in the study of micromechanical behaviour of granular media. While most existing DEM studies have employed spherical particles for easy manipulation of particles contacts and for better numerical performance, real granular particles, such as sand grains, are rarely perfectly spherical in shape. Nonspherical shape of real particles may lead to particle interlocking and hence different kinematic behaviour of the particle system. While it is difficult and expensive to simulate the real particle geometry in DEM, an alternative approach is to consider the nonspherical particle interlocking attributable to the resistance to the rolling of contacted particles, which will be employed in this study. We shall employ a three dimensional DEM code to investigate the shear behavior of granular materials. We employ a linear force-displacement contact law wherein the normal stiffness k_n and the tangential stiffness k_s are set equally to be $k_n/r = k_s/r$ = 100 MPa, where r is the radius of the particle. The interparticle sliding is assumed to be governed by Coulomb's friction law with a sliding frictional coefficient $\mu = 0.5$. Regarding the rolling resistance, we consider the following law governs the contact moment M_r and the relative rolling angle between particles θ_r through a rolling stiffness k_r:

$$M_r = -k_r\theta_r, \quad |M_r| \le M_{\max} = \mu_r f_n l \qquad (1)$$

where we assume $k_r = k_s r^2$, $r = r_1 r_2/(r_1+r_2)$, with r_1 and r_2 being the radii of the two contact particles, respectively (see also [1]). The threshold to contact moment in Eq. $(1)_2$ is similar to the Coulomb's friction law, where μ_r is the rolling resistance coefficient, f_n is the normal force and l is the characteristic length which is related to the particle radii and the penetration depth between particles (see Fig. 1). It is normally safe to assume that the penetration depth of the two particles is about 2% of the equivalent radius r. l can then be estimated to be $0.05r$. In this way, we may judge whether or not relative rolling will occur at a contact. A cubic packing of polydisperse spherical particles is considered. The cubic box is formed by six rigid frictionless walls. A total number of 31,769 particles with radii ranged from 0.2 mm to 0.6 mm are then randomly generated in the cubic. The size distribution of particles approximately obeys a power law. After desired number of particles are generated in the cubic box, isotropic consolidation is then conducted on the packing, which is accomplished by two stages with varied frictional coefficients. Packings with an initial void ratio around 0.647 are generated for the subsequent undrained shear.

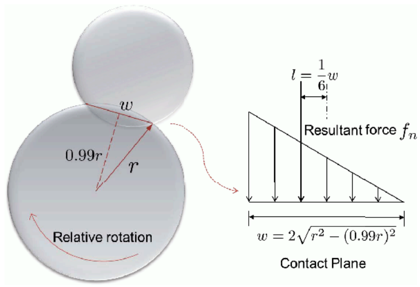

Figure 1. Contacting particles with tendency of relative rolling and triangular distribution of normal force

RESULTS AND DISCUSSION

The effects of rolling resistance are examined through a series of undrained tests. Since only dry particles are involved, the undrained condition is imposed by constraining the total volume of the sample to be constant during the shear. Similar treatment has been employed in [2]. We consider four cases of rolling resistance μ_r = 0.0, 0.1, 0.3 and 0.5 (corresponding to *UR0*, *UR1*, *UR3* and *UR5*, respectively, as discussed in the sequel). The responses of samples with different rolling resistance coefficients μ_r are shown in Fig. 2. Consideration of rolling resistance of particles essentially makes a sample stronger than otherwise. The shear strength of a sample increases steadily with the increase of μ_r. Note that UR0 is actually the case of free rolling which experiences obvious liquefaction. The introduction of rolling resistance may indeed help an otherwise liquefiable sample to develop appreciable resistance to liquefaction. Samples with μ_r = 0.1, 0.3 and 0.5 all regain strength after an obvious phase transformation state and approach the critical state.

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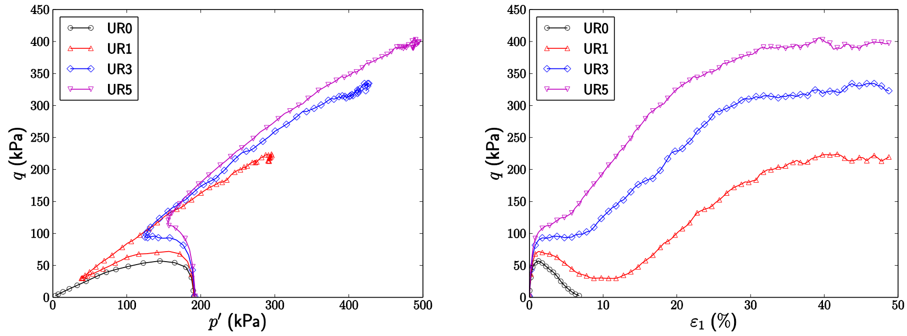

(a) (b)

Figure 2. Undrained responses of specimens with different rolling resistance. (a) Loading path; (b) stress-strain relation.

The following analytical stress-force-fabric correlation between the shear strength of a granular assembly and the various sources of anisotropy has been found (c.f. [3])

$$\eta = \frac{q}{p'} = \frac{2}{5}\left(a_c + a_d + a_n + \frac{3}{2}a_t\right) \tag{2}$$

where q and p' denote, respectively, the deviatoric stress and mean effective stress. a_c, a_d, a_n and a_t represent the second invariants of the deviatoric anisotropy tensor associated with fabric, branch vector, normal contact force and tangential contact force, respectively. Validation of the relation in Eq. (2) by our DEM simulation is presented in Fig. 3. While our previous study has found a good correlation between the analytical expression in Eq. (2) with DEM results without considering rolling resistance, the same conclusion still hold in the presence of rolling resistance as observed in Fig. 3 (also Fig. 2b). This is reasonable since rolling resistance will not change the static force equilibrium state of the assembly, based on which the analytical relationship in Eq. (2) has been developed. It is noticed, however, the employment of rolling resistance changes the shear strength of the material appreciably. A higher value of μ_r generally leads to a higher stress ratio during the shear. Indeed, the visualization of force chain network formed in the system (see Fig. 4) indicates the rolling resistance makes the interparticle contacts of a sample in the horizontal direction (perpendicular to shear) much harder to be detached and thus facilitates the formation of stronger lateral propping, which likely is able to support higher strong force chains in the vertical direction – it is thus reasonable to expect a much stronger material behaviour. The observation is consistent with the bimodal theory proposed in [4].

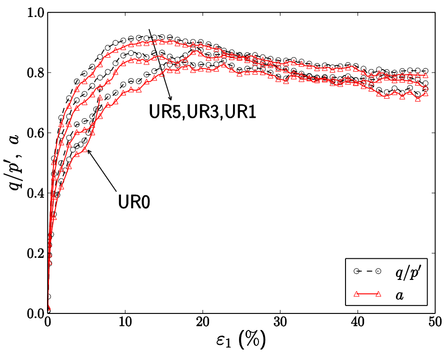

Figure 3. Correlation of the analytical stress-force-fabric relationship in Eq. (2) with DEM simulations

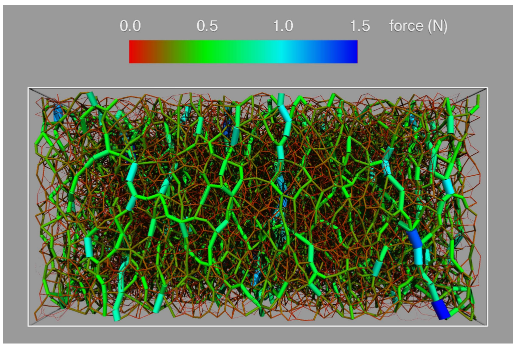

Figure 4. Strongly anisotropic force chain network formed during the shear of a DEM specimen

CONCLUSION

A micromechanical study for granular materials under confined shear has been presented using DEM. The effect of nonspherical particle shape is considered equivalently by introducing a rolling resistance model into the DEM code. It is observed that rolling resistance generally leads to stronger material response. Visualization of the anisotropic force chain network provides physically-based explanations for the observation. Good correlation between the analytical stress-force-fabric relationship with the DEM simulations still holds in consideration of rolling resistance.

References

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