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EFFECT OF CRACK ORIENTATION AND CONNECTIVITY ON EFFECTIVE PERMEABILITY OF MICROCRACKED SOLIDS

Chunsheng Zhou*[a)], Wei Wang*
*School of Civil Engineering, Harbin Institute of Technology, Harbin, 150090, CHINA

Summary For quasi-brittle materials like rock and concrete, cracks play an important role in its permeability. Orientation and inter-connection of cracks, which make microstructure obviously different from the one with no-overlapping inclusions separately embedded in matrix implied in classic composite theory, are significant features of microcracked solids. To capture the influence of orientation and inter-connection on permeability, interaction direct derivative (IDD) method is adopted for damaged solids with no inter-connection. Furthermore, IDD are extended to apply for partly inter-connecting case by introducing quantified connectivity. The influence of orientation on permeability of damaged solids, with connectivity or not, is solved by IDD method as well as numerical simulation. It is indicated that orientation and connectivity make much contribution to permeability. The shielding effect, which makes permeability over-estimated by IDD, becomes more remarkable with higher crack density and more centralized orientation.

INTRODUCTION

The influence of micro-cracking damage on permeability is currently an important issue in the field of mass transport research [1] and engineering applications [2]. Effective permeability as well as mechanical properties is rather sensitive to specific microstructure of cracks, which are the most common defects in damaged solids. The damage process of quasi-brittle materials like concrete and rock are composed of the nucleation of new microcracks, orienting propagation, clustering and connecting of existing ones. Accurate prediction of permeability requires to properly consider crack fraction, spatial distribution, possible clustering and connectivity etc. of crack phase [3]. It is reported that permeability of cracked concrete shows an important dependence on cracking orientation and connecting degree [4]. Though various approximation schemes were proposed to evaluate effective properties of composite material, it is incorrect to apply them directly to estimate permeability of damaged solids with inter-connected microcracks due to their exclusive implied assumption of matrix-inclusion structure without overlapping. To the best knowledge of the authors, the influence of orientation and connectivity on permeability have been rarely studied and published.

IDD ESTIMATE OF PERMEAILITY

Based on the same root of inclusion-matrix-composite model, a micromechanical scheme, the effective self-consistent scheme (ESCS), and a simplified version, the interaction direct derivative (IDD) have been proposed [5]. This scheme has an explicit structure with clear physical explanations for all involved components and presents analytical solutions for both effective elastic and other linear physical properties. Based on this scheme, a published study [6] submitted by the same author have derived the analytical permeability of cracked media and further extended this model consistently to consider the effect of limited crack connectivity. The permeability increment $\boldsymbol{H}$ of effective medium and permeability fluctuations $\boldsymbol{H}_i$ of inclusion ω_i are defined as $\boldsymbol{H} = \boldsymbol{S} - \boldsymbol{S}_0,\ \ \boldsymbol{H}_i = \boldsymbol{S}_i - \boldsymbol{S}_0$ where $\boldsymbol{S}$ is the permeability tensor and the permeability for matrix phase, ith inclusion and effective medium are labelled by 0, i and none respectively. Then, IDD estimate gives the effective permeability increment $\boldsymbol{H}^{\mathrm{idd}}$ as

$$\boldsymbol{H}^{\mathrm{idd}} = \left(\boldsymbol{I} - \sum_i \boldsymbol{H}_i^{\mathrm{d}} \cdot \boldsymbol{\Omega}_{Di}^{0}\right)^{-1} \cdot \boldsymbol{H}^{\mathrm{d}} \tag{1}$$

where $\boldsymbol{I}$, $\boldsymbol{\Omega}_{Di}^{0}$ and $\boldsymbol{H}^{\mathrm{d}}$ are unit tensor, eigenstiffness of matrix atmosphere and dilute estimate respectively [5].
For most quasi-brittle materials like concrete and rock, the permeability of even small cracks are several magnitude higher than porous matrix. So assuming the permeability of crack inclusions is infinite compared to that of matrix is reasonable. Density ρ is defined as $\rho = \sum a_i^2/A$ for a large enough representative area A (a_i is the half-length of ith crack) and elliptical matrix atmosphere is adopted for 2D cases mostly concerned here. In 2D global coordinate system defined by two orthogonal unit vector $\boldsymbol{g}_{1,2}$, the IDD estimate $\boldsymbol{H}^{\mathrm{idd}}$ of permeability is

$$\boldsymbol{H}^{\mathrm{idd}} = \left(\boldsymbol{I} - \pi\rho\gamma_D \boldsymbol{G}\right)^{-1} \pi\rho \boldsymbol{S}_0 \cdot \boldsymbol{G} \tag{2}$$

where γ_D is a quantity determined by ρ and matrix atmosphere [6] and second-rank tensor $\boldsymbol{G}$ denotes

$$\boldsymbol{G} = \mathrm{E}\left(\cos^2\theta\right)\boldsymbol{g}_1\boldsymbol{g}_1 + \mathrm{E}\left(\sin\theta\cos\theta\right)\left(\boldsymbol{g}_1\boldsymbol{g}_2 + \boldsymbol{g}_2\boldsymbol{g}_1\right) + \mathrm{E}\left(\sin^2\theta\right)\boldsymbol{g}_2\boldsymbol{g}_2 \tag{3}$$

For partly connected microcracks, the connectivity ϕ is quantified as

$$\phi = 1 - \sum_{i=1}^{r} a_i^2 / \sum_{j=1}^{t} a_j^2 \tag{4}$$

[a)] Corresponding author. E-mail: ZhouChunsheng.HIT@gmail.com

where t is the total number of microcracks and r is the number of isolated ones. IDD estimate can be extended to account partly inter-connected cracks by adopting effective density ρ' instead of ρ in Eq. (2), $\rho' = (1 + \beta\phi)\,\rho$ where β is a constant related to ρ and ϕ. It is also reported that the orientation angle θ observes normal distribution for quasi-brittle materials [7], $\theta \sim \mathcal{N}\left(0, \sigma_\theta^2\right)$ where σ_θ is noted as standard deviation. In this case, tensor $\boldsymbol{G}$ can be further arranged as

$$\boldsymbol{G} = \left[1/2 + \exp\left(-2\sigma_\theta^2\right)/2\right] \boldsymbol{g}_1\boldsymbol{g}_1 + \left[1/2 - \exp\left(-2\sigma_\theta^2\right)/2\right] \boldsymbol{g}_2\boldsymbol{g}_2 \tag{5}$$

σ_θ indicates the scattering of crack orientation, so parameter λ can be introduced as orienting degree, $\lambda = \sigma_\theta/\frac{\pi}{2}$. The larger λ is, the more uniform is the crack orientation. If $\lambda = 0$, all cracks are parallel. The influence of orienting degree λ on permeability can be explicitly defined from Eq. (2) and Eq. (5). For three different cracking densities $\rho = 0.3, 0.6, 0.9$, the influence of orienting degree λ on permeability component $S^{11,22}$ is shown in Fig. 1. The orienting degree λ have great influence on main component $S^{11,22}$ of permeability only when $\lambda < 1$.

NUMERICAL SIMULATION

Numerical tests on a properly designed representative volume element (RVE) is a reliable way to verify estimate scheme [2]. With the aid of Monte-Carlo algorithm, a well-developed RVE generation of 2D crack network with given density ρ and connectivity ϕ have been proposed [6]. In this paper, 160mm length square is chose and crack length $2a$ is supposed to observe log-normal distribution [8], $2a \sim \ln\mathcal{N}\left(2.983, 0.157^2\right)$, and $\mathrm{E}\,(2a) = 20$, $\mathrm{Var}\,(2a) = 10$. The main component of permeability $S^{11,22}$ for $\lambda = 1, 1/2, 1/3$, $\phi = 0, 0.5$ are solved numerically [6]. From the previous results [6], coefficient β is supposed to be linear with ρ, $\beta = 1.2\rho$ for $\phi = 0.5$ as $\rho \in [0, 1.0]$. The numerical results as well as IDD estimate are shown in Figure 2 and 3. From them one can see that the IDD estimate of S^{22} agrees well with numerical results while the over-estimate of S^{11} becomes greater with rising density ρ and centralized orientation (reducing λ). This is due to the stronger shielding effect of cracks as they approach closer and orienting degree gets higher.

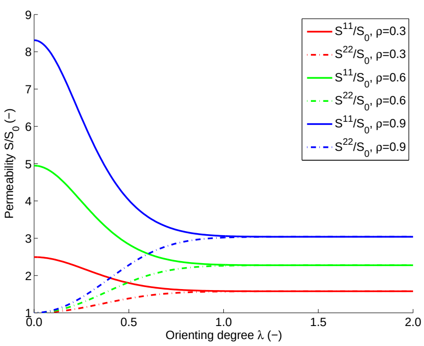

Figure 1. The influence of orienting degree λ on permeability S (constant ρ).

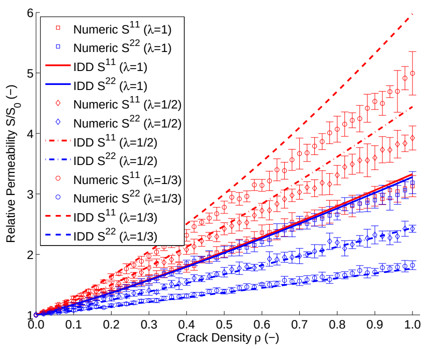

Figure 2. The relationship between crack density ρ and permeability S ($\phi = 0$).

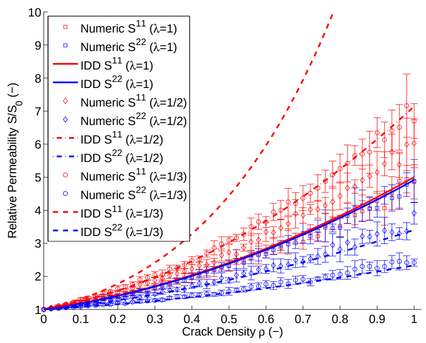

Figure 3. The relationship between crack density ρ and permeability S ($\phi = 0.5$).

CONCLUSIONS

IDD method and its extended version are employed to estimate effective permeability of micro-cracked solids with connectivity or not and the effect of crack orientation is focused. It is concluded that orienting degree has remarkable influence on permeability especially for high density cases. IDD method can capture the interaction of cracks with low density and orienting degree. Otherwise the permeability of damaged solids will be over-estimated due to the strong shielding effect.

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