STABILITY OF THE STATE-DEPENDENT DELAY MODEL OF TURNING WITH DEPTH OF CUT VARIATION

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Summary The effect of variation of the depth of cut due to tool vibration in the tangential direction on the stability of a state-dependent delay model of turning has been investigated in this paper. Significant increase in regions with stable cutting has been observed.

INTRODUCTION

Insperger *et al.* [1] proposed an accurate model of the regenerative effect during turning involving a state-dependent delay (wherein the delay is determined by a combination of the workpiece rotation and tool vibration). Insperger *et al.* [2] observed that the state-dependency of the delay marginally increases the stable regions of oiperation and further showed using a numerical continuation technique that there is a transition from subcritical to supercritical bifurcation as the feed-rate is increased. Wahi [3] confirmed these observations using an analytical study using the method of multiple scales. In this paper, we include the variation in the depth of cut resulting from tangential vibrations. This results in a significant increase in operating conditions with stable cutting.

MATHEMATICAL MODEL AND ITS ANALYSIS

Assuming compliance in both x- and y-direction as shown in Fig. 1, we get a 2 degree of freedom (DOF) model ([1, 2])

$$m\ddot{x}(t) + c_x\dot{x}(t) + k_x x(t) = K_x w(t) C(t)^q\,, \tag{1}$$

$$m\ddot{y}(t) + c_y\dot{y}(t) + k_y y(t) = K_y w(t) C(t)^q\,, \tag{2}$$

where m is the mass of the tool, c_x and c_y are the damping parameters, k_x and k_y are the tool stiffness while K_x and K_y are the cutting coefficients in the x and y-directions, respectively and $w(t)$ and $C(t)$ are the instantaneous chip width and chip thickness respectively, and q is the cutting force exponent (a measure of the nonlinearity in the cutting force). Following [1, 2], the instantaneous chip thickness is

$$C(t) = v\tau + y(t-\tau) - y(t)\,,$$

where v is the speed of the feed and the regenerative delay τ between the present and the previous cut is determined using

$$R\Omega\tau = 2R\pi + x(t) - x(t-\tau)\,,$$

where Ω is the spindle speed in rad/s and R is the radius of the workpiece. The instantaneous chip width $w(t)$ can be obtained as

$$w(t) = w_0 - R\left(1 - \sqrt{1 - \left(\frac{x(t)}{R}\right)^2}\right)\,,$$

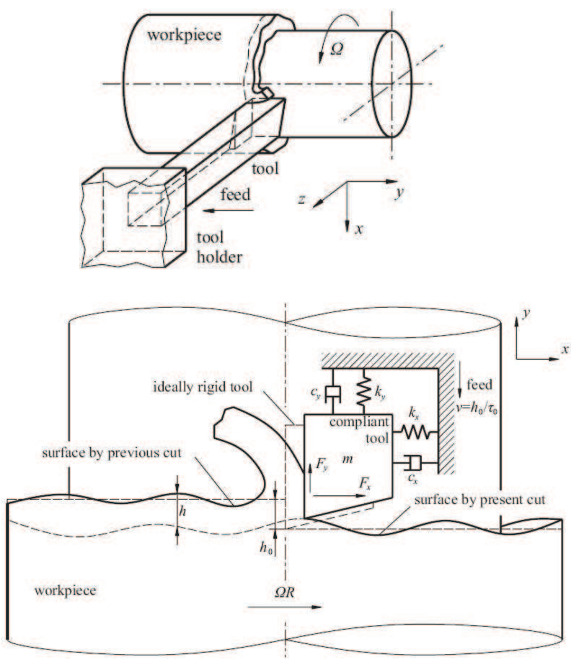

Figure 1. A 2 DOF model for turning.

where w_0 is the nominal depth of cut. Next introducing the cutting force ratio $k_r = F_y/F_x = K_y/K_x$ and noting that a substitution of $x(t) = y(t)/k_r$ in Eq. (1) reduces it identically to Eq. (2) if $k_x = k_y = k$ and $c_x = c_y = c$. Therefore, for the special case of the damping and stiffness properties being identical in the two directions, we effectively have a single DOF system (see [3]). Next non-dimensionalizing time as $\tilde{t} = \omega_n t$ with the natural frequency $\omega_n = \sqrt{k/m}$ and tool displacement as $y(t) = v\tau_0\tilde{y}(t)$ with $\tau_0 = 2\pi/\Omega$ as the mean delay and dropping the tilde we get

$$\ddot{y}(t) + 2\,\zeta\,\dot{y}(t) + y(t) = K_1\rho^{q-1}\left(wn - 1 + \sqrt{1 - \left(\frac{2\pi\rho y(t)}{k_r}\right)^2}\right)\left(\frac{\tau}{\tau_0} + y(t-\tau) - y(t)\right)^q\,, \tag{3}$$

where $\zeta = \dfrac{c}{2m\omega_n}$, $K_1 = \dfrac{K_y R(2\pi R)^{q-1}}{m\omega_n^2}$ is the dimensionless cutting force coefficient, $\rho = \dfrac{v\tau_0}{2\pi R}$ is the dimensionless

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feed rate, and $wn = w_0/R$ is the non-dimensional nominal depth of cut. The time-delay τ is determined by

$$\frac{\tau}{\tau_0} = 1 + \frac{\rho}{k_r}\left(y(t) - y(t-\tau)\right) . \tag{4}$$

The final equation for tool chatter including the chip width variation and regenerative effect is

$$\ddot{y}(t) + 2\,\zeta\,\dot{y}(t) + y(t) = K_1\rho^{q-1}\left(wn - 1 + \sqrt{1 - \left(\frac{2\pi\rho}{k_r}\right)^2 y(t)^2}\right)\left(1 + \frac{k_r - \rho}{k_r}\left(y(t-\tau) - y(t)\right)\right)^q . \tag{5}$$

The equilibrium solution of Eq. (5) with $\bar{\tau} = \tau_0$ is

$$\bar{y} = K_1\rho^{q-1}\left(\frac{k_r^2(wn-1)}{(4\pi^2\rho^{2q}K_1^2 + k_r^2)} + \frac{k_r}{(4\pi^2\rho^{2q}K_1^2 + k_r^2)}\sqrt{k_r^2 - 4\pi^2\rho^{2q}K_1^2 wn^2 + 8\pi^2\rho^{2q}K_1^2 wn}\right).$$

To get the linearized equation governing the stability of this equilibrium solution (stable cutting), we substitute $y(t) = \bar{y} + \eta(t)$ with $\eta(t) \ll 1$ in Eq. (5) such that nonlinear terms in $\eta(t)$ can be neglected. This results in

$$\ddot{\eta}(t) + 2\,\zeta\,\dot{\eta}(t) + \eta(t) = q\bar{y}\frac{k_r - \rho}{k_r}\left(\eta(t-\tau_0) - \eta(t)\right) + \frac{4\pi^2\rho^{q+1}K_1\bar{y}}{k_r\sqrt{k_r^2 - 4\pi^2\rho^2\bar{y}^2}}\eta(t) . \tag{6}$$

The second term on the right hand side is the contribution of the variable chip width to the linearized equation. To obtain the stability boundary corresponding to Eq. (6), we first obtain the characteristic equation by assuming a solution for $\eta(t) = \eta_0 e^{\lambda t}$ and substituting $\lambda = \pm i\omega$ corresponding to Hopf bifurcation. The resulting complex equation is separated into real and imaginary parts which can be solved for the cutting parameters wn and τ_0 (which is related to the non-dimensional spindle speed Ω/ω_n with ω as a parameter.

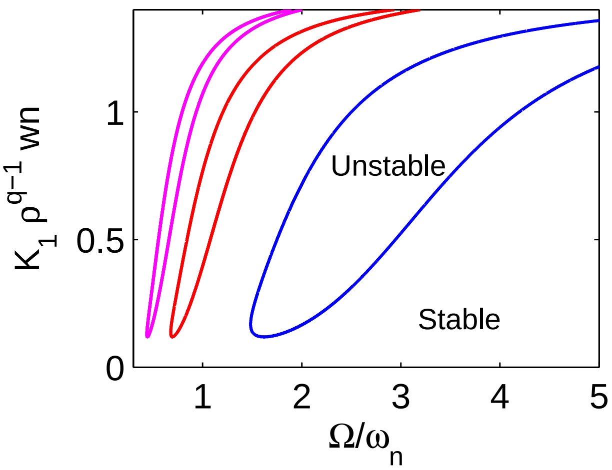

Figure 2. First few stability lobes/islands for a typical set of parameter values which do not correspond to practical applications.

A typical stability charts for this system for some extreme values of the parameters which are not encountered in practical cutting operations is depicted in Fig. 2. The first few stability lobes have been shown in this figure which clearly demonstrates that the adjacent stability lobes no longer intersect giving rise to stability islands instead. Stability charts for other probably more realistic values from the practical standpoint show similar trends but they no longer remain isolated and intersect at larger depths of cut. However, the bending of the stability boundaries which causes re-entry into a stable region with an increase in the depth of cut is present for all set of parameter values. This observation points toward some interesting insights into the dynamics of turning which can be obtained by a detailed nonlinear study of the system which is currently underway. We also note that vibrations in tangential directions are associated with other phenomenon like variable effective rake angle as well as variable cutting velocity which can add further complexity to the model. The effect of these on the stability of steady cutting requires a detailed understanding of the dependence of the cutting coefficient on these parameters and has been left for a future study.

CONCLUSIONS

Variation in the depth of cut due to vibrations in the tangential direction has a significant effect on the stability of the cutting process in turning. There is an increase in the stable regime and the effect on the criticality of bifurcation requires further investigation. The change in the stability of the system with inclusion of variable chip width highlights the need to incorporate effects like variable rake angle and cutting velocity for a more comprehensive model for turning operation.

References

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